中央研究院數學研究所
招考研習員試題暨錄取名單

1. (a) Find the minimal polynomial for the matrix:

$$\begin{bmatrix} 3 & 1 & -1 \\ 0 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}.$$

   (b) Let A and B be $n \times n$ matrices over the field F. Do the matrices AB and BA have the same characteristic polynomial? Do they have the same minimal polynomial?

2. Show that the polynomial ring $\mathbf{Z}[x]$ is an unique factorization domain.

3. (a) Is the function $f(z) = \bar{z}$ differentiable? Where? (z complex variable)

   (b) Let u be a real-valued harmonic function on $\mathbf{C}$. For what functions f is the function $f(u)$ also harmonic? Explain it!

   (c) Let $f : \mathbf{C} \to \mathbf{C}, \mathbf{C}$ is the complex plane, be an analytic function. We know that f maps the imaginary axis into $D = \{z \in \mathbf{C}, | z |= 1/2\}$, and $f(1) = f(-1)$. What should the function f be? Explain it!

   (d) Is the differential

$$ds = \frac{| dz |}{1-| z |^2} \qquad (| z |< 1)$$

   invariant with the group of fractional linear transformations which transforms the circle $| z |< 1$ into itself? Explain it!

4. Prove $T(x) = \int_0^\infty e^{-t}t^{x-1}dt$ converge for $x > 0$. And

$$\frac{T(x)}{e^{-x}x^{x-\frac{1}{2}}} \longrightarrow \sqrt{2\pi} \quad as \quad x \to \infty.$$

(Note: $\int_{-\infty}^{\infty} e^{-\frac{t^2}{2}}dt = \sqrt{2\pi}$).

5. (a) Prove $\lim_{n\to\infty}\int_0^1 f(x)\sin(2\pi nx)dx = 0$ for f which is continuous on $[0,1]$.

   (b) Show that if f, g are continuous functions on $[0,\infty)$ with $f(x) = f(x+1)$ and $g(x) = g(x+1)$, then

$$\lim_{n\to\infty}\int_0^1 f(x)g(nx)dx = \int_0^1 f(x)dx\int_0^1 g(x)dx.$$

6. Let $f(x)$ be a positive continuous function on $(0,1)$ such that $\int_0^1 f(x)dx < \infty$. Show that for any $\varepsilon > 0$, there exists a $\delta > 0$ such that $\int_a^b f(x)dx < \varepsilon$ for any $0 < a, b < 1$ with $(b-a) < \delta$.

7. Let (a,b) be a finite interval in R. Prove or disprove that if f is uniformly continuous on (a,b) then f is bounded on (a,b).

錄取名單:

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