

8202 微分與積分的等式 (韓良信提供)

將 0, 1, 2, ..., 7 填入下面的空格,

$$\begin{aligned} & \frac{d}{dx} \log (e^{\square x} + \sin \square x - x^{\square}) \Big|_{x=\square} \\ &= \square \\ &= \frac{1}{100} \int_{\square}^{\square} x^{\square} dx \end{aligned}$$

解答：(郭仲成提供)

$$\begin{aligned} & \text{設 } \frac{d}{dx} \log (e^{\square x} + \sin \square x - x^{\square}) \Big|_{x=\square} \\ &= \square \\ &= \frac{1}{100} \int_{\square}^{\square} x \, dx \\ &= \frac{d}{dx} \log (e^{ax} + \sin bx - x^c) \Big|_{x=d} = e \\ &= \frac{1}{100} \int_g^h x^f dx \end{aligned}$$

$$\therefore e \in \{0, 1, 2, \dots, 7\}$$

$$\therefore \frac{1}{100} \int_g^h x^f dx \in Z^+$$

$$\Rightarrow h > g, \text{ 且 } f \neq 0, 1$$

由計算知 $f = 3$ 且 $g = 1, h = 7$, 恰好

$$\text{能將一百除盡, 且 } \frac{1}{100} \int_{g=1}^{h=7} x^3 dx$$

$$= \frac{1}{100} \left. \frac{x^4}{4} \right|_1^7 = 6$$

$$\therefore \frac{d}{dx} \log (e^{ax} + \sin bx - x^c) \Big|_{x=d}$$

$$= \frac{ae^{ax} + b \cos bx - cx^{c-1}}{e^{ax} + \sin bx - x^c} \Big|_{x=d}$$

$$\therefore e = 6$$

$$\therefore ae^{ad} + b \cos bd \in Q \text{ (有理數)}$$

$$\therefore d = 0$$

$$\therefore \frac{ae^{ad} + b \cos bd - cd^{c-1}}{e^{ad} + \sin bd - d^c}$$

$$= \begin{cases} a + b - 1 & \text{當 } c = 1 \\ \text{or } a + b & \text{當 } c \neq 1 \end{cases}$$

故 $a + b - 1$ 不合 (\because 前已使用過 1)

$$\therefore a + b = e = 6$$

取 $a = 4$ 且 $b = 2$ or 取 $a = 2$ 且 $b = 4$
兩組。

$$\therefore c = 5$$

$$\text{即 } \frac{d}{dx} \log (e^{2x} + \sin 4x - x^5) \Big|_{x=0}$$

$$= \boxed{6} = \frac{1}{100} \int_1^7 x^3 dx$$

$$\text{和 } \frac{d}{dx} \log (e^{4x} + \sin 2x - x^5) \Big|_{x=0}$$

$$= \boxed{6} = \frac{1}{100} \int_1^7 x^3 dx$$

兩組解。

當然如果 0, 1, 2, ..., 7 可重覆填入
空格, 則有很多明顯的解, 在此討論毫無意義。