Preface

This special issue contains the first of three parts of the Proceedings of the Taipei Winter School and Conference on Representation Theory held on December 16-23, 2010. The remaining two parts will appear in the following two issues of the Bulletin of the Institute of Mathematics Academia Sinica N.S.. These three issues are edited by Profs. Shun-Jen Cheng and Weiqiang Wang.

Representation theory is an important field of research in modern mathematics, in which one attempts to realize symmetries (understood as groups or other algebraic structures) in terms of matrices and to gain new insights from such constructions. There have been numerous connections and applications of representation theory to algebraic combinatorics, geometry, mathematical physics, and other disciplines. Even within its own discipline, representation theory is very diverse, as one can work with finite- or infinite-dimensional groups or algebras over different fields (of characteristic zero or prime characteristic).

Such was the background when the Taipei Winter School in Representation Theory was held on December 16–19, followed by the Taipei Conference on Representation Theory on December 20–23, 2010. Two mini-courses were presented during the winter school. The one by Mark Shimozono, "Affine Schubert calculus", addressed recent applications of representation theory to combinatorics of symmetric functions in the framework of homology and cohomology of affine Grassmannians. The other one by Weiqiang Wang (with supplementary lectures by Jinkui Wan), "Combinatorial representation theory of the spin symmetric group", described spin representation theory of symmetric groups which was first developed by Schur in 1911 and its rich combinatorial structures.

Understandably, Taipei (pronounced as "Type A") is the favorite of many representation theorists in the conference. Encouraged by the high level of talks given at the conference and the enthusiastic responses of the participants, it was decided to prepare some proceedings. The diversity of the topics in the papers of the proceedings (which are published in three volumes of the Bulletin) is a reflection of the many stimulating lectures which touched various active research directions of representation theory.

The paper of Naihuan Jing and Robert Ray studies the quantum symmetric spaces for quantum general linear groups modulo symplectic groups and their spherical functions which appear as the quantum analogue of zonal spherical polynomials. In his contribution Nanhua Xi shows that Lusztig's

a-function of a Coxeter group is bounded if the Coxeter group has a complete graph (i.e., any two vertices are joined) and the cardinalities of finite parabolic subgroups of the Coxeter group have a common upper bound.

Aaron Lauda's expository article explains how planar diagrammatics naturally arise in the study of categorified quantum groups with a focus on the categorification of quantum $\mathfrak{sl}_2$. It, in particular, includes a proof that cyclotomic quotients of the nilHecke algebra are isomorphic to the matrix rings over the cohomology ring of Grassmannians, and an interpretation of fake bubbles using symmetric functions. The survey paper of Anthony Licata and Alistair Savage presents an overview of various graphical categorifications of the Heisenberg algebra and its Fock space representation which are intimately related to the geometry of Hilbert schemes.

George Lusztig studies the geometry of the G-orbits on a certain variety X_w associated to an elliptic element of the Weyl group of a connected reductive group G. The results of the paper further enhance the view that the variety X_w can be regarded as a limit case as $q \mapsto 1$ of the celebrated Deligne-Lusztig variety associated to w (defined over $\mathbb{F}_q$).

In his contribution Alexander Molev constructs a basis for each covariant tensor representation (also called polynomial representation) of the general linear Lie superalgebra $\mathfrak{gl}(m|n)$, which generalizes the classical Gelfand-Tsetlin basis. The main role in the construction is played by the fact that the subspace of $\mathfrak{gl}(m)$-highest vectors in any finite-dimensional irreducible representation of $\mathfrak{gl}(m|n)$ carries a structure of an irreducible module over the Yangian $Y(\mathfrak{gl}(n))$. Minoru Wakimoto's paper addresses bi-Hamiltonian structures of Poisson W-algebras of minimal nilpotent elements and shows that there exist two kinds of bi-Hamiltonian pairs, which are generalization of the KdV and the Harry-Dym hierarchies associated to the Virasoro Poisson vertex algebra.

In their paper, Jon Calson and Dan Nakano show that there are only finitely many endotrivial modules of a finite group scheme at each dimension. For a finite group G there was a (stronger) result of Puig that the group of endotrivial modules is finitely generated. Claus Ringel's paper presents a full proof that any torsionless-finite Artin algebra has representation dimension at most 3, and shows that the representation dimension is adjusted very well to forming tensor products of algebras. Along the way he obtains a wealth of examples of Artin algebras that are torsionless-finite or of large representation dimensions.

In the lecture notes on spin representation theory of symmetric groups, Jinkui Wan and Weiqiang Wang provide an exposition of recent developments on spin analogues of several classical constructions including Frobenius characteristic map, Schur duality, coinvariant algebras, Kostka-Foulkes polynomials, and Young's seminormal form construction.

We thank all the speakers for their enlightening lectures in a very memorable conference and thank all the authors for their fine contributions to the proceedings. Finally, we thank Li-Wu Chen who was indispensable in the preparation and organization of the winter school and conference.

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