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ON STRONGLY EQUIPRIME $\Gamma-$ NEAR RINGS

BY

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Abstract

In this paper we obtain some equivalent conditions for strongly equiprime $\Gamma-$ near rings N and the strongly equiprime radical $\mathcal{P}_{se}(N)$ coincides with $\mathcal{P}_{se}(L)^{+}$where $\mathcal{P}_{se}(L)$ is the strongly equiprime radical of left operator near-ring L of N.

1. Introduction

The concept of $\Gamma-$ near ring, a generalization of both the concepts near-ring and $\Gamma-$ ring was introduced by Satyanarayana [12]. Later, several authors such as Satyanarayana [11], Booth and Booth Groenewald [2, 3, 4] studied the ideal theory of $\Gamma-$ near rings. In this paper we obtain some equivalent conditions for strongly equiprime $\Gamma-$ near rings N and the strongly equiprime radical $\mathcal{P}_{se}(N)$ to coincide with $\mathcal{P}_{se}(L)^{+}$where $\mathcal{P}_{se}(L)$ is the strongly equiprime radical of left operator near-ring L of N.

2. Preliminaries

In this section we recall certain definitions needed for our purpose.

Definition 2.1. A Γ - near ring is a triple $(N,+,\Gamma)$, where

(i) $(N,+)$ is a (not necessarily abelian) group;
(ii) Γ is a non-empty set of binary operations on N such that for each $\gamma\in\Gamma$, $(N,+,\gamma)$ is a right near -ring and;

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(iii) $(x\gamma y)\,\mu z = x\gamma\,(y\mu z)$ for all $x, y, z \in N$ and $\gamma, \mu \in \Gamma$.

Γ-near rings generalize near-rings in the sense that every near-ring N is a Γ-near ring with $\Gamma = \{\cdot\}$, where $\cdot$ is the multiplication defined on N. Another example is the following : Let X and G be a non empty set and an additive group respectively. Let $N = M(X, G)$ and let $\Gamma = M(G, X)$, where $M(A, B)$ denotes the set of all mappings from A into B. Then N is a Γ- near ring with the operations pointwise addition and composition of mappings.

Definition 2.2. Let N be a $\Gamma-$near ring, then a normal subgroup I of $(N, +)$ is said to be

(i) left ideal (right ideal) if $a\alpha\,(b + i) - a\alpha b \in I \quad \forall\ a, b \in N, i \in I$ and $\alpha \in \Gamma$,
(ii) right ideal if $i\alpha a \in I \quad \forall\ i \in I, a \in N$ and $\alpha \in \Gamma$,
(iii) ideal if it is both a left and a right ideal of N.

Definition 2.3. A subgroup I of $(N, +)$ is said to be left (right) $\Gamma-$subgroup of N if $N\Gamma I \subseteq I\,(I\Gamma N \subseteq I)$.

I is said to be $\Gamma-$ subgroup if it is both a left and a right $\Gamma-$subgroup.

Definition 2.4. Let N be a $\Gamma-$ near ring. Let $\mathcal{L}$ be the set of all mappings of N into itself which act on the left. Then $\mathcal{L}$ is a right near-ring with operations pointwise addition and composition of mappings. Let $x \in N$and $\alpha \in \Gamma$. We define the mapping $[x, \alpha] : N \to N$ by $[x, \alpha]\, y = x\alpha y$ $\forall\, y \in N$. The sub near-ring L of $\mathcal{L}$ generated by the set $\{[x, \alpha]\,/x \in N, \alpha \in \Gamma\}$ is called the left operator near-ring of N. If $I \subseteq L$, then

$$I^{+} = \{x \in N/\,[x, \alpha] \in I\ \forall\ \alpha \in \Gamma\}.$$

If $J \subseteq N$, $J^{+'} = \{\ell \in L/\ell x \in J\ \forall\ x \in N\}$. It is shown in [3] that I is an ideal in L implies I^{+} is an ideal in N and J is an ideal in N implies $J^{+'}$ is an ideal in L.

A right operator near-ring R of N is defined analogously to the definition of L. Let $\mathcal{R}$ be the left near-ring of all mappings of N in to itself which act on the right. If $\gamma \in \Gamma$,$y \in N$, we define $[\gamma, y] : N \to N$ by $x\,[\gamma, y] = x\gamma y$ for all $x \in N$. R is the sub near-ring of $\mathcal{R}$ generated by the set $\{[\gamma, y]\,/\gamma \in \Gamma, y \in N\}$.

Definition 2.5. An element x of a Γ-near ring N is called distributive if $x\alpha\,(a+b) = x\alpha a + x\alpha b$ for all $a, b \in N$ and $\alpha \in \Gamma$. If all the elements of a Γ-near ring N are distributive, then N is said to be a distributive Γ-near ring.

Definition 2.6. A $\Gamma-$ near ring N is said to be zero symmetric if $a\gamma 0 = 0 \;\; \forall\, a \in N, \gamma \in \Gamma$.

Definition 2.7. An element m in a $\Gamma-$ near ring N is said to be a left non-zero divisor if $m\alpha x = 0$ implies that $x = 0$ for any $\alpha \in \Gamma$. An element n is said to be a right non-zero divisor $y\alpha n = 0$ implies that $y = 0$ for any $\alpha \in \Gamma$. An element in a $\Gamma-$ near ring is said to be a non-zero divisor if it is both left and right non-zero divisor of N.

Definition 2.8. Let N be a Γ-near ring with left operator near-ring L. If $\sum\limits_i [d_i, \delta_i] \in L$ has the property that $\sum\limits_i d_i\delta_i x = x \;\forall\, x \in N$, then $\sum\limits_i [d_i, \delta_i]$ is called a left unity for N. A strong left unity for N is an element $[d, \delta]$ of L such that $d\delta x = x \;\forall\, x \in N$.

Definition 2.9. An ideal I of a $\Gamma-$ near ring N is called a completely prime ideal of N if $a, b \in N$ and $\alpha \in \Gamma, a\alpha b \in I$ implies $a \in I$ or $b \in I$.

Definition 2.10. An ideal I of a $\Gamma-$ near ring N is said to be prime if for any two ideals A, B of N , $A\Gamma B \subseteq I$ implies $A \subseteq I$ or $B \subseteq I$.

3. Strongly Equiprime Γ− Near Rings

In this section we shall prove that some equivalent conditions for strongly equiprime $\Gamma-$ near rings.

Definition 3.1. Let N be a $\Gamma-$ near ring. N is said to be strongly equiprime if for each $a \neq 0 \in N$, there exists finite subsets F of N and Δ of Γ respectively, such that $a\gamma f\mu x = a\gamma f\mu y \;\; \forall\; f \in F, \gamma, \mu \in \Delta$ imply $x = y$ $\forall\, x, y \in N$. Here F is called an insulator for a.

Definition 3.2. Let N be a $\Gamma-$ near ring, then

$$N_c = \{n \in N / n\gamma 0 = n \,\forall\, \gamma \in \Gamma\} = \left\{n \in N / \;\forall\, n' \in N : n\gamma n' = n \,\forall\, \gamma \in \Gamma\right\}$$

is called the constant part of N.

The notations $< a >$ and $< a >_\Gamma$ will denote respectively the ideal and the $\Gamma-$ subgroup generated by a in N.

Definition 3.3. Let N be a $\Gamma-$ near ring and $I =< a >_\Gamma$ be a $\Gamma-$ subgroup of N. For any $x \in I, \gamma \in \Gamma$, there exists $x_1, x_2, \ldots, x_n \in I$ such that $x_n = x$ and for each $1 \leq i \leq n$ one of the following holds:

$$\begin{aligned} x_i &= ma \text{ for some } m \in \mathbb{Z} \\ x_i &= x_j \pm x_k \text{ for some } j, k < i \\ x_i &= r\gamma x_j \text{ for some } r \in N, j < i \\ x_i &= x_j \gamma r \text{ for some } r \in N, j < i. \end{aligned}$$

This sequence $x_1, x_2, \ldots, x_n$ is called a generating sequence for x.

Theorem 3.4. *Let N be a strongly equiprime $\Gamma-$near ring. Then the left operator near-ring L is strongly equiprime.*

Proof. Suppose N is strongly equiprime. We shall prove that L is strongly equiprime. Let $0 \neq l \in L$. Then there exists $x \in N$ such that $lx \neq 0$. Since N is strongly equiprime, there exist $f_1, f_2, \ldots f_n \in N$ and $\gamma_1, \gamma_2, \ldots \gamma_n \in \Gamma$ such that $y, z \in N$ and $(lx)\gamma_i f_j \gamma_k y = (lx)\gamma_i f_j \gamma_k z$ for all $1 \leq i, j, k \leq n$ implies $y = z$. Let $G = \{[x\gamma_i f_j, \gamma_k] : 1 \leq i, j, k \leq n\}$. Let $l_1, l_2 \in L$ and suppose that $lgl_1 = lgl_2$ for all $g \in G$. Then $lgl_1 y = lgl_2 y$ for all $y \in N$. Hence $l[x\gamma_i f_j, \gamma_k]l_1 y = l[x\gamma_i f_j, \gamma_k]l_2 y$ and so $(lx)\gamma_i f_j \gamma_k(l_1 y) = (lx)\gamma_i f_j \gamma_k(l_2 y)$ for all $1 \leq i, j, k \leq n$. Hence $l_1 y = l_2 y$ for all $y \in N$ and so $l_1 = l_2$. Hence L is strongly equiprime. $\square$

Definition 3.5. A $\Gamma-$ near ring N is said to be a left weakly semiprime $\Gamma-$near ring if $[x, \Gamma] \neq 0 \; \forall \; x \neq 0 \in N$.

Note that if N is a distributive $\Gamma-$ near-ring, then the elements of L are expressible in the form $\sum_i [x_i, \alpha_i]$ and also N is strongly equiprime if and only if it is strongly prime, that is, if $0 \neq x \in N$, then there exist finite subsets F and Δ of N and Γ respectively, such that $x\Delta F\Delta y = 0$ implies $y = 0$, for all $y \in N$.

Theorem 3.6. *Let N be a distributive, left weakly semiprime $\Gamma-$ near ring having no zero divisor, then N is strongly equiprime if and only if L is strongly equiprime.*

Proof. Suppose L is strongly equiprime. We shall prove that N is strongly equiprime. Let $x \neq 0 \in N$. Since L is strongly equiprime, there exist finite subsets

$$F = \left\{ \sum_{j=1}^{n} [y_{jk}, \beta_{jk}] / k = 1, 2, \ldots, m \right\} \qquad \text{(say)}$$

of N and Δ of Γ respectively, such that

$$[x, \gamma]\, f \ell_1 = [x, \gamma]\, f \ell_2 \;\forall\; f \in F, \gamma \in \Delta \text{ implies } \ell_1 = \ell_2 \;\forall\; \ell_1, \ell_2 \in L. \qquad (1)$$

Consider $F' = \{y_{jk}\beta_{jk}x / j = 1, 2, \ldots, n, k = 1, 2, \ldots, m\}$. Our claim is that F' is an insulator for x. Let $y, z \in N, \gamma, \mu \in \Delta$ such that

$$x\gamma y_{jk}\beta_{jk}x\mu y = x\gamma y_{jk}\beta_{jk}x\mu z \;\forall\; j = 1, 2, \ldots, n; k = 1, 2, \ldots, m.$$

We shall prove that $y = z$. Now

$$x\gamma y_{jk}\beta_{jk}x\mu y = x\gamma y_{jk}\beta_{jk}\mu z \;\forall\; j = 1, 2, \ldots, n; k = 1, 2, \ldots, m$$

implies

$$\begin{aligned} [x\gamma y_{jk}\beta_{jk}x\mu y - x\gamma y_{jk}\beta_{jk}\mu z, \Gamma] &= 0 \;\forall\; j = 1, 2, \ldots, n; k = 1, 2, \ldots, m. \\ \text{i.e., } [x\gamma y_{jk}\beta_{jk}x\mu y - x\gamma y_{jk}\beta_{jk}\mu z, \delta] &= 0 \;\forall\; \delta \in \Gamma \\ &\quad \text{and } \forall\; j = 1, 2, \ldots, n;\; k = 1, 2, \ldots, m. \end{aligned}$$

Hence

$$\begin{aligned} [x\gamma y_{jk}\beta_{jk}x\mu y, \delta\,] - [x\gamma y_{jk}\beta_{jk}\mu z, \delta\,] &= 0 \;\forall\; \delta \in \Gamma \\ &\quad \text{and } \forall\; j = 1, 2, \ldots, n; k = 1, 2, \ldots, m. \end{aligned}$$

Then

$$\begin{aligned} [x\gamma y_{jk}\beta_{jk}x\mu y, \delta\,] &= [x\gamma y_{jk}\beta_{jk}x\mu z, \delta\,] \;\;\forall\; \delta \in \Gamma \\ &\quad \text{and } \forall\; j = 1, 2, \ldots, n; k = 1, 2, \ldots, m, \\ \text{i.e., } [x, \gamma]\, [y_{jk}, \beta_{jk}]\, [x\mu y, \delta] &= [x, \gamma]\, [y_{jk}, \beta_{jk}]\, [x\mu z, \delta] \end{aligned}$$

for all $\delta \in \Gamma$ and for all $j = 1, 2, \ldots, n; k = 1, 2, \ldots, m$. Therefore,

$$[x, \gamma] \sum_{j=1}^{n} [y_{jk}, \beta_{jk}] [x\mu y, \delta] = [x, \gamma] \sum_{j=1}^{n} [y_{jk}, \beta_{jk}] [x\mu z, \delta] \ .$$

By $(1), [x\mu y, \delta] = [x\mu z, \delta]$, i.e., $[x\mu y - x\mu z, \delta] = 0$ for all $\delta \in \Gamma$. Since N is left weakly semiprime, $x\mu y - x\mu z = 0$. Hence $x\mu (y - z) = 0$. Since N has no zero divisor, $y - z = 0$ and hence $y = z$.

Converse part follows from Theorem 3.4. □

Proposition 3.7. *Let N be a $\Gamma-$ near ring. Then the following statements are equivalent:*

(1) *N is strongly equiprime;*
(2) *Every non zero right $\Gamma-$ subgroup of N contains a finite subset F such that $x, y \in N, f\gamma x = f\gamma y \ \forall \ f \in F, \gamma \in \Gamma$ implies $x = y$;*
(3) *Every non zero $\Gamma-$ subgroup of N contains a finite subset F such that $x, y \in N, f\gamma x = f\gamma y \ \ \forall \ f \in F, \gamma \in \Gamma$ implies $x = y$.*

Proof. (1) $\Rightarrow$ (2): Let $I \neq 0$ be right $\Gamma-$ subgroup of N and $a \neq 0 \in I$. Then there exist finite subsets F and Δ of N and Γ respectively, such that $a\alpha f\beta x = a\alpha f\beta y$ for all $f \in F$ and $\alpha, \beta \in \Delta$ implies $x = y$ for all $x, y \in N$. Let $G = \{a\alpha f / \alpha \in \Delta, f \in F\}$. Then G is a finite subset of I and it satisfies our required result.

(2) $\Rightarrow$ (3): Obvious.

(3) $\Rightarrow$ (1): First we show that N is zero symmetric. For if N is not zero symmetric, then $N_c \neq 0$, and so $n\alpha x = n\alpha y = n \ \forall \ n \in N_c, \alpha \in \Gamma$. Hence N_c contains no finite subset with the required property. This contradiction shows that N is zero symmetric. Let $a \neq 0 \in N$. Suppose that $a\Gamma N = 0$, then $< a > \Gamma N = 0$. Since N is zero symmetric, $< a >$ is a $\Gamma-$ subgroup of N and $< a > \neq 0$. Moreover $b\alpha x = b\alpha y = 0 \ \forall \ b \in < a >, x, y \in N, \alpha \in \Gamma$. Hence $< a >$ can not contain a finite subset F such that $x, y \in N, f\alpha x = f\alpha y \ \forall \ f \in F$ implies $x = y$. This contradiction shows that $a\Gamma N \neq 0$. Let $n \in N$ be such that $a\alpha n \neq 0$. Consider a $\Gamma-$ subgroup $I = < a >_\Gamma$ of N. Let Δ be a finite subset of Γ and let $f_1, f_2, \ldots, f_n \in I$ be such that $f_i\alpha x = f_i\alpha y, \forall \ \alpha \in \Delta, 1 \leq i \leq n$ implies $x = y \ \forall \ x, y \in N$. Let $f_{i1}, f_{i2}, \ldots, f_{im(i)}$ be a generating sequence for $f_i, 1 \leq i \leq n$. Each f_{ij}

contains factors of the form $a\alpha n$ or $(a\alpha n)\,\beta r\,\forall\,\alpha,\beta\in\Delta$ and for some $r\in N$. Let $G=\{n\}\cup\{n\alpha r/\,(a\beta n)\,\alpha r$ occurs in some $f_{ij}\}$. Suppose $x,y\in N$, $\alpha,\beta\in\Delta$ and that $(a\alpha g)\,\beta x=(a\alpha g)\,\beta y\ \ \forall\ g\in G$. It follows from the definition of generating sequence that $f_{ij}\beta x=f_{ij}\beta y\ \ \forall\ 1\le i\le n, 1\le j\le m\,(i)$ and in particular that $f_i\beta x=f_i\beta y\ \ \forall\ 1\le i\le n$. Hence $x=y$. Since G is finite, it is an insulator for a. Hence N is strongly equiprime. □

Definition 3.8.([5]) A $\Gamma-$ near ring N is said to be equiprime if $a,x,y\in N$ and $\alpha,\beta\in\Gamma, a\alpha n\beta x=a\alpha n\beta y$ for all $n\in N$ implies $a=0$ or $x=y$.

Note that

(1) Every equiprime Γ-near ring is zero symmetric.
(2) Every strongly equiprime Γ-near ring is equiprime .

Definition 3.9. Let N be a Γ-near ring and A be a subset of N. Then the right equalizer of A is the set

$$r_e\,(A)=\{(x,y)\in N\times N/a\alpha x=a\alpha y\ \forall\ a\in A,\alpha\in\Gamma\}\,.$$

Proposition 3.10. *Let N be an equiprime $\Gamma-$near ring which satisfies the d.c.c on right equalizers, then N is strongly equiprime.*

Proof. If $N=0$, then the result is trivial. So assume that $N\neq 0$. Let $I\neq 0$ be a $\Gamma-$ subgroup of N and $\mathcal{M}=\{r_e\,(F)\,/F$ is a finite subset of $I\}$. Since N satisfies the d.c.c. on the right equalizers, $\mathcal{M}$ contains a minimal element $E=r_e(F_0)$ say. We claim that $E=\{(x,x)\,/x\in N\}$. For if not, there exists $(x,y)\in E$ with $x\neq y$. Let $a\neq 0\in I$. Since N is equiprime, there exists $n\in N$ such that $a\alpha n\beta x\neq a\alpha n\beta y$, where $\alpha,\beta\in\Gamma$. Let $F_1=F_0\cup\{a\alpha n\}$. Then $F_1\subseteq I$ and since $F_0\subseteq F_1$,

$$r_e\,(F_1)\subseteq r_e\,(F_0)\,. \tag{2}$$

Moreover, since $a\alpha n\beta x\neq a\alpha n\beta y, (x,y)\notin r_e\,(F_1)$. But $(x,y)\in r_e\,(F_0)$. Hence the inclusion in (2) is strict. This contradicts the minimality of E. Hence $E=\{(x,x)\,/x\in N\}$. Thus if $x,y\in N, x\neq y$ implies $(x,y)\notin E$. Therefore there exists $f\in F_0$ such that $f\alpha x\neq f\alpha y$. It follows from Proposition 3.7 that N is strongly equiprime. □

Definition 3.11. If X is a subset of $N \times N$, then the left equalizer of X is the set $\ell_e(X) = \{a \in N / a\alpha x = a\alpha y \ \forall \ (x, y) \in X, \alpha \in \Gamma\}$.

Lemma 3.12. *Let N be a $\Gamma-$near ring and D a left equalizer in N. Then $D = \ell_e(r_e(D))$.*

Proof. Let $X \subseteq N \times N$ be such that $D = \ell_e(X)$. Let $d \in D, (x, y) \in r_e(D)$. Then $d\gamma x = d\gamma y$, for all $\gamma \in \Gamma$ and hence $d \in \ell_e(r_e(D))$. This implies that $D \subseteq \ell_e(r_e(D))$. A similar argument shows that $X \subseteq r_e(\ell_e(X))$. Hence $\ell_e r_e(\ell_e(X)) \subseteq \ell_e(X)$, i.e., $\ell_e(r_e(D)) \subseteq D$ and consequently $D = \ell_e(r_e(D))$. □

Lemma 3.13. *Let N be a $\Gamma-$near ring. Then N satisfies the d.c.c on right equalizers if and only if N satisfies the a.c.c. on left equalizers.*

Proof. Suppose N satisfies the d.c.c. on right equalizers. Let $D_1 \subseteq D_2 \subseteq \cdots$ be an ascending chain of left equalizers. Then $r_e(D_1) \supseteq r_e(D_2) \supseteq \cdots$ is a descending chain of the right equalizers. Then there exists $n \in N$ such that $r_e(D_n) = r_e(D_{n+1}) = \cdots$. Hence $\ell_e(r_e(D_n)) = \ell_e(r_e(D_{n+1})) = \cdots$, i.e., $D_n = D_{n+1} = \cdots$ by Lemma 3.12. The proof of the converse is similar. □

Corollary 3.14. *Every equiprime $\Gamma-$near ring N with a.c.c. on left equalizers is strongly equiprime.*

Proof. Suppose N is an equiprime $\Gamma-$ near ring with a.c.c. on left equalizers. By Lemma 3.13, N satisfies the d.c.c. on right equalizers. It follows from Proposition 3.10 that N is strongly equiprime. □

4. Strongly Equiprime Radicals of $\Gamma-$ Near Rings

In this section we shall prove that the strongly equiprime radical $\mathcal{P}_{se}(N)$ coincides with $\mathcal{P}_{se}(L)^+$ where $\mathcal{P}_{se}(L)$ is the strongly equiprime radical of left operator near-ring L of N.

Notation 4.1. For a $\Gamma-$ near ring N, the prime radical and the set of all nilpotent elements are denoted by $\mathcal{P}_0(N)$ and $\mathcal{N}(N)$ respectively.

Definition 4.2. An ideal I of a Γ-near ring N is said to be 2-primal if $\mathcal{P}_0\left(\frac{N}{I}\right) = \mathcal{N}\left(\frac{N}{I}\right)$.

A Γ-near ring N is called strongly 2-primal if every ideal I of N is 2-primal . If the zero ideal of N is 2-primal, then N is called 2-primal. This is equivalent to $\mathcal{P}_0(N) = \mathcal{N}(N)$.

The following theorem characterizes 2-primalness for ideals in Γ-near rings. The proof is a minor modification the of proof of the corresponding theorem in near-ring theory [1].

Theorem 4.3. *Let I be an ideal of a Γ-near ring N. Then*

(i) *I is a completely semiprime ideal if and only if I is both a semiprime and a 2-primal ideal.*
(ii) *If $N\Gamma I \subseteq I$, then the following are equivalent:*
   (a) *I is a completely prime ideal;*
   (b) *I is both a prime and a completely semiprime ideal;*
   (c) *I is both a prime and a 2-primal ideal.*

Lemma 4.4. *If a Γ-near ring N is strongly 2-primal, then every prime ideal of N is completely prime.*

Proof. It follows from Theorem 4.3. □

Definition 4.5. Let N be a Γ− near ring. An ideal P is said to be strongly equiprime if for each $a \notin P$, there exists finite subsets F and Δ of N and Γ respectively, such that $a\gamma f\mu x - a\gamma f\mu y \in P\ \forall\ f \in F$, and $\gamma, \mu \in \Delta$ implies $x - y \in P\ \forall\ x, y \in N$.

Proposition 4.6. *Let N be a Γ− near ring. If P is a strongly equiprime ideal of N, then $P^{+'} = \{\ell \in L/\ell x \in P\ \forall\ x \in N\}$ is a strongly equiprime ideal of L.*

Proof. Suppose P is a strongly equiprime ideal of N. We shall prove that $P^{+'}$is a strongly equiprime ideal of L. Let $0 \neq l \notin P^{+'}$. Then there exists $x \in N$ such that $lx \notin P$. Since P is strongly equiprime ideal, there exist $f_1, f_2, \ldots f_n \in N$ and $\gamma_1, \gamma_2, \ldots \gamma_n \in \Gamma$ such that $y, z \in N$ and $(lx)\gamma_i f_j \gamma_k y - (lx)\gamma_i f_j \gamma_k z \in P$ for all $1 \leq i, j, k \leq n$ implies $y - z \in P$. Let $G = \{[x\gamma_i f_j, \gamma_k] : 1 \leq i, j, k \leq n\}$. Let $l_1, l_2 \in L$ and suppose that $lgl_1 - lgl_2 \in P^{+'}$ for all $g \in G$. Then $(lgl_1 - lgl_2)y \in P$ for all $y \in N$. Hence $l[x\gamma_i f_j, \gamma_k]l_1 y - l[x\gamma_i f_j, \gamma_k]l_2 y \in P$ and so $(lx)\gamma_i f_j \gamma_k(l_1 y) - (lx)\gamma_i f_j \gamma_k(l_2 y) \in P$ for all

$1 \leq i,j,k \leq n$. Hence $l_1y - l_2y \in P$ for all $y \in N$ and so $l_1 - l_2 \in P^{+'}$. Hence $P^{+'}$ is strongly equiprime ideal of L. □

Proposition 4.7. *Let N be a distributive strongly 2-primal $\Gamma-$ near ring with a strong left unity, and Q a strongly equiprime ideal of L. Then*

$$Q^+ = \{x \in N / [x,\alpha] \in Q \ \forall\ \alpha \in \Gamma\}$$

is strongly equiprime ideal of N.

Proof. Suppose $0 \neq Q$ is a strongly equiprime ideal of L. We shall prove that $Q^+$is a strongly equiprime ideal in N. Let $x \notin Q^+$. Then there exists $\alpha \in \Gamma$ such that $[x,\alpha] \notin Q$. Since N is distributive and Q is a strongly equiprime ideal in L, there exists a finite subsets $F = \left\{ \sum_{j=1}^{n} [y_{jk}, \beta_{jk}] / k = 1,2,\ldots,m \right\}$ of N and Δ of Γ respectively, such that

$$[x,\alpha] \sum_{j=1}^{n} [y_{jk}, \beta_{jk}]\, \ell_1 - [x,\alpha] \sum_{j=1}^{n} [y_{jk}, \beta_{jk}]\, \ell_2 \in Q$$

for all $k = 1,2,\ldots m$ and $\alpha, \beta \in \Delta$ implies

$$\ell_1 - \ell_2 \in Q \ \ \forall\ \ell_1, \ell_2 \in L. \tag{3}$$

Consider the collection

$$F' = \{y_{jk}\beta_{jk}x / j = 1,2,\ldots n; k = 1,2,\ldots m\}.$$

Our claim is that F' is an insulator for x. Let $a, b \in N$,$\alpha, \beta \in \Delta$ such that

$$x\alpha y_{jk}\beta_{jk}x\beta a - x\alpha y_{jk}\beta_{jk}x\beta b \in Q^+ \ \ \forall\ j = 1,2,\ldots n; k = 1,2,\ldots m.$$

We shall prove that $a - b \in Q^+$. Now

$$x\alpha y_{jk}\beta_{jk}x\beta a - x\alpha y_{jk}\beta_{jk}x\beta b \in Q^+ \ \ \forall\ j = 1,2,\ldots n; k = 1,2,\ldots m$$

implies

$$[x\alpha y_{jk}\beta_{jk}x\beta a - x\alpha y_{jk}\beta_{jk}x\beta b, \gamma] \in Q$$

for all $\gamma \in \Gamma$ and for all $j = 1, 2, \ldots n; k = 1, 2, \ldots m$. Hence

$$[x\alpha y_{jk}\beta_{jk}x\beta a, \gamma] - [x\alpha y_{jk}\beta_{jk}x\beta b, \gamma] \in Q,$$

$$\text{i.e., } [x, \alpha] [y_{jk}, \beta_{jk}] [x\beta a, \gamma] - [x, \alpha] [y_{jk}, \beta_{jk}] [x\beta b, \gamma] \in Q$$

for all $\gamma \in \Gamma$ and for all $j = 1, 2, \ldots n; k = 1, 2, \ldots m$.

Therefore

$$\text{i.e.,} \quad [x, \alpha] \sum_{j=1}^{n} [y_{jk}, \beta_{jk}] [x\beta a, \gamma] - [x, \alpha] \sum_{j=1}^{n} [y_{jk}, \beta_{jk}] [x\beta b, \gamma] \in Q$$

for all $\gamma \in \Gamma$ and for all $k = 1, 2, \ldots, m$. By (3), $[x\beta a, \gamma] - [x\beta b, \gamma] \in Q$, i.e., $[x\beta a - x\beta b, \gamma] \in Q$ for all $\gamma \in \Gamma$. Hence $x\beta a - x\beta b \in Q^+$. This implies that $x\beta (a - b) \in Q^+$. Since Q is strongly equiprime in L, Q is prime in L. By [3, Proposition 3.3], Q^+ is prime in N. Since N is strongly 2-primal, Q^+ is completely prime in N. Hence $x\beta (a - b) \in Q^+$ and $x \notin Q^+$ implies $a - b \in Q^+$. Thus Q^+ is strongly equiprime ideal in N. □

Theorem 4.8. *Let N be a distributive strongly 2-primal $\Gamma-$ ring with a strong left unity. Then $\mathcal{P}_{se}(L)^+ = \mathcal{P}_{se}(N)$ where L is the left operator near-ring of N, $\mathcal{P}_{se}(N)$ is the strongly equiprime radical of N and $\mathcal{P}_{se}(L)$ is the strongly equiprime radical of L.*

Proof. Let P be a strongly equiprime ideal of L. Then by Proposition 4.7, P^+ is a strongly equiprime ideal of N. Moreover $(P^+)^{+\prime} = P$ by [2, Proposition 5]. Suppose Q is a strongly equiprime ideal in N, then by Proposition 4.6, $Q^{+\prime}$ is a strongly equiprime ideal in L and $\left(Q^{+\prime}\right)^+ = Q$ by [2, Proposition 5]. Thus the mapping $P \to P^+$ defines a 1-1 correspondence between the set of strongly equiprime ideals of L and N. Hence $\mathcal{P}_{se}(L)^+ = (\cap P)^+ = \cap P^+ = \mathcal{P}_{se}(N)$. □

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