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MAGNETOELASTIC SURFACE WAVES IN ELECTRICALLY CONDUCTING FIBRE-REINFORCED MEDIA

BY

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Abstract

This paper investigates the propagation of surface waves in fibre-reinforced electrically conducting elastic solid media permeated by a primary magnetic field. First, the theory of general surface waves has been derived and then this theory has been applied to study Rayleigh, Love and Stoneley type of waves, which are the special cases of the above surface waves. The frequency equations are found in every case. The results obtained in this paper may be considered as more general in the sense that some other important formulas investigated by different authors may be deduced from our result as special cases. Combined effect of magnetic field, electrical conductivity and the reinforcement of the medium on the propagation of Rayleigh and Love type of waves have been studied numerically with graphical representations. Numerical study of the Stoneley waves is not pursued due to its complicated nature. It is found that the presence of magnetic field in electrically conducting fibre-reinforced media modulates the Rayleigh wave velocity to a considerable extent. Though the fibre-reinforcement has a significant effect on the propagation of Love waves, magnetic field has no role to play in this case. The modulation of Love wave velocity due to fibre-reinforcement has also been studied numerically.

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1. Introduction

Interactions between strain and electromagnetic fields are largely being undertaken due to its various applications in many branches of science and technology. Development of magnetoelasticity also induces us to study various problems of geophysics, seismology and related topics. Bazer [3] made a survey of linear and non-linear wave motion in a perfect magnetoelastic medium. Without going into the details of the research work published so far in the fields of magnetoelasticity, magneto-thermo-elasticity, magneto-thermo-viscoelasticity we mention some recent papers [1], [9, 10], [13], [20], [29], [31, 32]. At the present time problems of propagation of waves in anisotropic media have been discussed by many others [6], [11], [12], [14], [17], [21], [27], [33].

It is known that surface waves play an important role in the study of earthquake, geophysics and geodynamics. Rayleigh waves, Love waves and Stoneley waves are the special cases of general surface waves. They are tightly connected with the earthquake spectrum analysis [23]. Rayleigh waves cause destruction to the structure owing to its slower attenuation of the energy than that of the body waves and the characteristic that it propagates along the surface. Recently P R Sengupta and S Nath [25] investigated surface waves in fibre-reinforced anisotropic elastic media. Comments on "surface waves in fibre-reinforced anisotropic elastic media" by P R Sengupta and S Nath may be seen in a paper presented by Sarvajit Singh [26].

The superiority of fibre-reinforced composite materials over other structural materials attracted many authors [7, 8], [16], [25], [28] to study different type of problems in this field. Fibre-reinforced composite concrete structures are used due to their low weight and high strength. Two important components namely concrete and steel of a reinforced medium are bound together as a single unit so that there can be no relative displacement between them i.e. they act together as a single anisotropic unit so long as they remain in the elastic condition. The artificial structures on the surface of the earth are excited during an earthquake, which gives rise to violent vibrations [24] in some cases. Engineers and architects are in search of such materials of the structures that will resist the oscillatory vibration. Again, the earth is placed in its own magnetic field and most concrete construction near the surface of the earth includes steel reinforcing at least nominally. The propagation of waves depends upon the ground vibration and the physical properties of the structure. Thus, the role of the propagation of surface waves in a magnetoelastic fibre-reinforced medium can not be neglected in any way. In most of the previous investigations, the effect of reinforcement has been neglected. The idea of continuous self-reinforcement at every point of an elastic solid may be obtained from the investigation presented by Spencer [30] and Belfield et al. [4]. Hashin and Rosen [16] introduced the elastic moduli for fibre-reinforced materials. Reflection of waves at the boundaries of fibre-reinforced media has been studied by several authors [8, 9, 10], [28]. Merodio and Ogden [18] investigated mechanical response of fibre-reinforced incompressible non-linearly elastic solids. Crampin et al. [11] and Crampin [12] considered propagation of surface waves in anisotropic media.

The above-mentioned authors have not discussed the combined effect of magnetic field, electrical conductivity and fibre-reinforcement on the propagation of surface waves in anisotropic media. In this paper starting from the formulation of general surface waves we have discussed Rayleigh, Love and Stoneley waves as special cases. Numerical calculation and graphs have been presented for Rayleigh and Love waves only. It is marked that Love waves are not effected by magnetic field though the fibre-reinforcing has a significant role to play on such waves. It is believed that the investigations presented in this paper have not been studied so far.

2. Basic Equations and Formulations of the Problem:

We consider a two dimensional model [fig.1] consisting of two homogeneous anisotropic fibre-reinforced elastic solid semi-infinite media M and M_1 with different elastic and reinforcement parameters. They are perfectly welded in contact at a plane interface. Let us take an orthogonal cartesian axes $oxyz$ with the origin o at the common plane boundary and oz points vertically upwards into the medium M $(z \geq 0)$. Each of the media M $(z \geq 0)$ and M_1 $(z \leq 0)$ extends to an infinitely great distance from the plane horizontal boundary surface of separation $z = 0$. The two media are permeated into a uniform magnetic field **H.**

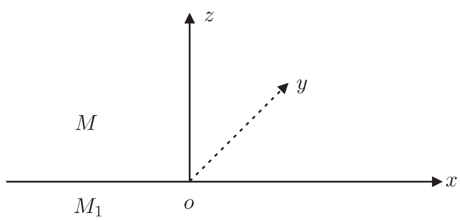

Figure 1. Schematic diagram of the problem.

It is assumed that the waves travel in the positive direction of the x-axis in such a way that the disturbance is largely confined to the neighborhood of the boundary and at any instant, all particles have equal displacements in any direction parallel to *oy*. In view of the above assumptions, the waves may be considered as surface waves and all partial derivatives with respect to y are zero. We assume that an induced magnetic field $\mathbf{h} = (0, h, 0)$ and an induced electric field $\mathbf{E}$ are developed due to the application of initial magnetic field $\mathbf{H} = (0, H_0, 0)$.

For a slowly moving homogeneous electrically conducting elastic solid medium, the linear equations of electrodynamics in a simplified form may be presented as [20], [31]

$$\left.\begin{aligned} \nabla \times \mathbf{h} &= \mathbf{J} + \varepsilon_0 \dot{\mathbf{E}} \\ \nabla \times \mathbf{E} &= -\mu_0 \dot{\mathbf{h}} \\ \nabla \cdot \mathbf{h} &= 0 \\ \mathbf{E} &= -\mu_0 \left(\dot{\mathbf{u}} \times \mathbf{H} \right) \end{aligned}\right\} \tag{2.1}$$

where ∇ is the Hamilton's operator, ε_0 is the electric permeability, μ_0 is the magnetic permeability and $\mathbf{u}$ is the dynamic displacement vector. Overdot ($^\bullet$)represents derivative with respect to time. Here we ignore the small effect of temperature gradient on the current density vector $\mathbf{J}$. The deformation is supposed to be small and the dynamic displacement vector is actually measured from a steady state deformed position.

The components of the magnetic intensity vector in the medium are

$$H_x = 0,\ H_y = \mathrm{H}_0 + h\,(x, z, t)\,,\ H_z = 0 \tag{2.2}$$

Since the current density vector **J** must be parallel to **E**, we have the components of **J** as

$$J_x = J_1,\ J_y = 0,\ J_z = J_3 \tag{2.3}$$

As the electric intensity vector is normal to both the magnetic intensity and the displacement vectors, its components are

$$E_x = E_1,\ E_y = 0,\ E_z = E_3 \tag{2.4}$$

The equations governing the propagation of small elastic disturbances including only the interactions of mechanical and electromagnetic fields [9]

$$\sigma_{ij,j} + \mu_0 \left(\mathbf{J} \times \mathbf{H}\right)_i = \rho \frac{\partial^2 u_i}{\partial t^2}, \qquad (i, j = 1, 2, 3) \tag{2.5}$$

where σ_{ij} are the components of stress tensor, ρ is the mass density and **u**$= u_i = (u, v, w)$.

Using (2.5) the equations of motion for the present problem may be written in the following forms

$$\left.\begin{array}{l} \frac{\partial \sigma_{11}}{\partial x} + \frac{\partial \sigma_{13}}{\partial z} + \mu_0 \left(\mathbf{J} \times \mathbf{H}\right)_1 = \rho \frac{\partial^2 u}{\partial t^2} \\ \frac{\partial \sigma_{21}}{\partial x} + \frac{\partial \sigma_{23}}{\partial z} + \mu_0 \left(\mathbf{J} \times \mathbf{H}\right)_2 = \rho \frac{\partial^2 v}{\partial t^2} \\ \frac{\partial \sigma_{31}}{\partial x} + \frac{\partial \sigma_{33}}{\partial z} + \mu_0 \left(\mathbf{J} \times \mathbf{H}\right)_3 = \rho \frac{\partial^2 w}{\partial t^2} \end{array}\right\} \tag{2.6}$$

The constitutive equations for a fibre-reinforced linearly elastic anisotropic medium with respect to the reinforcement direction a are [4], [30]

$$\begin{aligned} \sigma_{ij} = {} & \lambda e_{kk}\delta_{ij} + \alpha \left(a_k a_m e_{km}\delta_{ij} + e_{kk} a_i a_j\right) + 2\left(\mu_L - \mu_T\right)\left(a_i a_k e_{kj} + a_j a_k e_{ki}\right) \\ & + \beta a_k a_m e_{km} a_i a_j \end{aligned} \tag{2.7}$$

where e_{ij} are components of infinitesimal strain; λ, μ_T are elastic constants; α, β, μ_L are reinforcement parameters and $\boldsymbol{a} = (a_1, a_2, a_3)$; $a_1^2 + a_2^2 + a_3^2 = 1$. We choose the fibre direction as $\boldsymbol{a} = (1,\ 0,\ 0)$.

The relevant components of stress tensor are

$$\left.\begin{aligned}
\sigma_{11} &= (\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta)\, e_{11} + (\lambda + \alpha)\, e_{22} + (\lambda + \alpha)\, e_{33}\\
\sigma_{22} &= (\lambda + \alpha)\, e_{11} + (\lambda + 2\mu_T)\, e_{22} + \lambda e_{33}\\
\sigma_{33} &= (\lambda + \alpha)\, e_{11} + \lambda e_{22} + (\lambda + 2\mu_T)\, e_{33}\\
\sigma_{12} &= 2\mu_L e_{12}\\
\sigma_{13} &= 2\mu_L e_{13}\\
\sigma_{23} &= 2\mu_T e_{23}
\end{aligned}\right\} \tag{2.8}$$

where

$$2e_{ij} = u_{i,j} + u_{j,i} \tag{2.9}$$

Using equation (2.1) one may find the components of $\mu_0\,(\mathbf{J}\times\mathbf{H})$. Thus introducing (2.1) and (2.8) into the equations in (2.6) one obtains the concerned equations of motion as follows:

$$\left.\begin{aligned}
&A_{11}\tfrac{\partial^2 u}{\partial x^2} + B_2\tfrac{\partial^2 w}{\partial x\partial z} + B_1\tfrac{\partial^2 u}{\partial z^2} - \mu_0 H_0\tfrac{\partial h}{\partial x} - \varepsilon_0\mu_0^2 H_0^2\tfrac{\partial^2 u}{\partial t^2} = \rho\tfrac{\partial^2 u}{\partial t^2}\\
&B_1\tfrac{\partial^2 v}{\partial x^2} + B_3\tfrac{\partial^2 v}{\partial z^2} = \rho\tfrac{\partial^2 v}{\partial t^2}\\
&A_{22}\tfrac{\partial^2 w}{\partial z^2} + B_2\tfrac{\partial^2 u}{\partial x\partial z} + B_1\tfrac{\partial^2 w}{\partial x^2} - \mu_0 H_0\tfrac{\partial h}{\partial z} - \varepsilon_0\mu_0^2 H_0^2\tfrac{\partial^2 w}{\partial t^2} = \rho\tfrac{\partial^2 w}{\partial t^2}
\end{aligned}\right\} \tag{2.10}$$

where

$$\begin{aligned}
&A_{11} = \lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta, \quad A_{22} = \lambda + 2\mu_T,\\
&B_1 = \mu_L, \;\; B_2 = \lambda + \alpha + \mu_L, \;\; B_3 = \mu_T
\end{aligned}$$

From equations (2.1) we may get the expression for h as

$$h = -H_0\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) \tag{2.11}$$

Insertion of (2.11) in (2.10) yields the following dynamical equations of motion valid for the medium M as

$$(A_{11}+\mu_0 H_0^2)\frac{\partial^2 u}{\partial x^2} + \left(B_2 + \mu_0 H_0^2\right)\frac{\partial^2 w}{\partial x\partial z} + B_1\frac{\partial^2 u}{\partial z^2} = \left(\rho + \varepsilon_0\mu_0^2 H_0^2\right)\frac{\partial^2 u}{\partial t^2} \tag{2.12}$$

$$B_1\frac{\partial^2 v}{\partial x^2} + B_3\frac{\partial^2 v}{\partial z^2} = \rho\frac{\partial^2 v}{\partial t^2} \tag{2.13}$$

$$(A_{22}+\mu_0 H_0^2)\frac{\partial^2 w}{\partial z^2} + \left(B_2 + \mu_0 H_0^2\right)\frac{\partial^2 u}{\partial x\partial z} + B_1\frac{\partial^2 w}{\partial x^2} = \left(\rho+\varepsilon_0\mu_0^2 H_0^2\right)\frac{\partial^2 w}{\partial t^2} \tag{2.14}$$

Since the media M and M_1 posses different elastic and reinforcement parameters and both the media are permeated into a single uniform magnetic field, the equations of motion valid for the medium M_1 may be written as

$$(A'_{11}+\mu_0 H_0^2)\frac{\partial^2 u'}{\partial x^2}+(B'_2+\mu_0 H_0^2)\frac{\partial^2 w'}{\partial x\partial z}+B'_1\frac{\partial^2 u'}{\partial z^2}=(\rho'+\varepsilon_0\mu_0^2 H_0^2)\frac{\partial^2 u'}{\partial t^2} \quad (2.12a)$$

$$B'_1\frac{\partial^2 v'}{\partial x^2}+B'_3\frac{\partial^2 v'}{\partial z^2}=\rho'\frac{\partial^2 v'}{\partial t^2} \quad (2.13a)$$

$$(A'_{22}+\mu_0 H_0^2)\frac{\partial^2 w'}{\partial z^2}+(B'_2+\mu_0 H_0^2)\frac{\partial^2 u'}{\partial x\partial z}+B'_1\frac{\partial^2 w'}{\partial x^2}=(\rho'+\varepsilon_0\mu_0^2 H_0^2)\frac{\partial^2 w'}{\partial t^2} \quad (2.14a)$$

where (u', v', w') are the displacement components for the medium M_1 and the primes $(')$ signify the corresponding material constants for the same.

3. Boundary Conditions

The stress continuity conditions across the boundary $z = 0$ are given by

$$\left(\sigma_{3j}+\sigma_{3j}^E\right)_{z\to 0+}-\left(\sigma'_{3j}+\sigma'^E_{3j}\right)_{z\to 0-}=0, \quad j=1,2,3. \quad (3.1)$$

where σ_{3j},σ'_{3j} are the stress components for M and M_1 respectively and σ_{3j}^E, σ'^E_{3j} are the corresponding components of Maxwell stress tensor; [9]

$$\sigma_{ij}^E=\mu_0\left(H_i h_j+H_j h_i-H_k h_k\delta_{ij}\right)$$

in which $H_i = (H_1, H_2, H_3)$ =initial constant magnetic field **H,** $h_j = (h_1, h_2, h_3)$ =induced magnetic field **h** and δ_{ij} =Kronecker delta.

In the present problem both the media are permeated by a uniform magnetic field **H.** As considered by many authors [1, 9, 13, 22, 29] in their different magnetoelastic problems, the continuity of Maxwell stress tensor along the boundary surface of separation is assumed in this problem also and hence

$$\left[\sigma_{3j}^E\right]_{z\to 0+}=\left[\sigma'^E_{3j}\right]_{z\to 0-}, \; j=1,2,3 \quad (3.1a)$$

Insertion of (3.1a) in (3.1) leads to

$$\left[\sigma_{3j}\right]_{z\to 0+}=\left[\sigma'_{3j}\right]_{z\to 0-}$$

Thus, at the interface of the two magneto elastic fibre-reinforced medium the stress continuity conditions (3.1) reduces to

$$\sigma_{13} = \sigma'_{13}, \qquad \sigma_{23} = \sigma'_{23}, \qquad \sigma_{33} = \sigma'_{33} \quad \text{on } z = 0. \tag{3.2}$$

where symmetric condition of stress components has been applied.

Again the components of displacements at the boundary surface $z = 0$ between two media M and M_1 must be continuous at all places and for all times i.e.

$$u = u', \qquad v = v', \qquad w = w' \quad \text{on } z = 0. \tag{3.3}$$

4. Solution of the Problem

Befitting the actual situation of the problem, we seek solutions of (2.12), (2.13) and (2.14) valid for the medium M as [5]

$$(u, v, w) = \{\hat{u}(z),\ \hat{v}(z),\ \hat{w}(z)\} \exp\{i\omega(x - ct)\} \tag{4.1}$$

where $\hat{u}(z)$,$\hat{v}(z)$,$\hat{w}(z)$ are functions of z only and $\frac{2\pi}{\omega}$ represents wave length of the harmonic waves traveling forward with speed c.

Analogous solutions of (2.12a), (2.13a) and (2.14a) valid for the medium M_1 $(-\infty < z \le 0)$ may be taken as

$$\left(u', v', w'\right) = \left\{\hat{u}'(z),\ \hat{v}'(z),\ \hat{w}'(z)\right\} \exp\{i\omega(x - ct)\} \tag{4.2}$$

in which $\hat{u}'(z)$,$\hat{v}'(z)$,$\hat{w}'(z)$ are functions of z only.

Substituting (4.1) into the equations (2.12) and (2.14), we arrive at the following equations:

$$\left[B_1 D^2 + \omega^2 \left\{\left(\rho + \varepsilon_0 \mu_0^2 H_0^2\right) c^2 - \left(A_{11} + \mu_0 H_0^2\right)\right\}\right] \hat{u} + i\omega \left(B_2 + \mu_0 H_0^2\right) D\hat{w} = 0 \tag{4.3}$$

$$\left[\left(A_{22} + \mu_0 H_0^2\right) D^2 + \omega^2 \left\{\left(\rho + \varepsilon_0 \mu_0^2 H_0^2\right) c^2 - B_1\right\}\right] \hat{w} + i\omega \left(B_2 + \mu_0 H_0^2\right) D\hat{u} = 0 \tag{4.4}$$

where $D \equiv \frac{d}{dz}$.

From equations (4.3) and (4.4) we get the following equations determining $\hat{u}$ or $\hat{w}$ as

$$\left(D^2+\lambda_1^2\omega^2\right)\left(D^2+\lambda_2^2\omega^2\right)(\hat{u},\,\hat{w})=0 \tag{4.5}$$

where

$$\left.\begin{aligned}
&\lambda_1^2+\lambda_2^2=\frac{\begin{array}{c}\left(A_{22}+\mu_0H_0^2\right)\left\{\left(\rho+\varepsilon_0\mu_0^2H_0^2\right)c^2-\left(A_{11}+\mu_0H_0^2\right)\right\}\\ +B_1\left\{\left(\rho+\varepsilon_0\mu_0^2H_0^2\right)c^2-B_1\right\}+\left(B_2+\mu_0H_0^2\right)^2\end{array}}{B_1\left(A_{22}+\mu_0H_0^2\right)}\\
&\text{and}\\
&\lambda_1^2\lambda_2^2=\frac{\left\{\left(\rho+\varepsilon_0\mu_0^2H_0^2\right)c^2-B_1\right\}\left\{\left(\rho+\varepsilon_0\mu_0^2H_0^2\right)c^2-\left(A_{11}+\mu_0H_0^2\right)\right\}}{B_1\left(A_{22}+\mu_0H_0^2\right)}
\end{aligned}\right\} \tag{4.6}$$

Now u, v, w, describe surface waves and as such they must be vanishingly small as $z\to\infty$. Hence in view of the equations (4.5) we take exponential solutions of (2.12) and (2.14) in the following forms:

$$\left.\begin{aligned}
u&=A\exp\left\{i\omega\left(-\lambda_1z+x-ct\right)\right\}+B\exp\left\{i\omega\left(-\lambda_2z+x-ct\right)\right\}\\
w&=A'\exp\left\{i\omega\left(-\lambda_1z+x-ct\right)\right\}+B'\exp\left\{i\omega\left(-\lambda_2z+x-ct\right)\right\}
\end{aligned}\right\} \tag{4.7}$$

where A, B, A', B' are constants.

Moreover, the solution of (2.13) may be expressed as

$$v=E\exp\left\{i\omega\left(-\lambda_3z+x-ct\right)\right\} \tag{4.8}$$

in which E is a constant and

$$\lambda_3^2=\frac{\rho c^2-B_1}{B_3} \tag{4.9}$$

Using (4.7) in (2.12) and (2.14) and equating the coefficients of $e^{-i\omega\lambda_1z}$ and $e^{-i\omega\lambda_2z}$ to zero we obtain

$$A'=k_1A,\quad B'=k_2B \tag{4.10}$$

where $k_i=\dfrac{B_1\lambda_i^2-\left(\rho+\varepsilon_0\mu_0^2H_0^2\right)c^2+\left(A_{11}+\mu_0H_0^2\right)}{\lambda_i\left(B_2+\mu_0H_0^2\right)}$, $i=1,2.$

Finally the displacement components for the medium M may be expressed as

$$\left.\begin{aligned} u &= A\exp\{i\omega(-\lambda_1 z + x - ct)\} + B\exp\{i\omega(-\lambda_2 z + x - ct)\} \\ v &= E\exp\{i\omega(-\lambda_3 z + x - ct)\} \\ w &= k_1 A\exp\{i\omega(-\lambda_1 z + x - ct)\} + k_2 B\exp\{i\omega(-\lambda_2 z + x - ct)\} \end{aligned}\right\} \quad (4.11)$$

Similarly, for the medium M_1 we derive

$$\left.\begin{aligned} u' &= C\exp\{i\omega(\lambda'_1 z + x - ct)\} + D\exp\{i\omega(\lambda'_2 z + x - ct)\} \\ v' &= E'\exp\{i\omega(\lambda'_3 z + x - ct)\} \\ w' &= k'_1 C\exp\{i\omega(\lambda'_1 z + x - ct)\} + k'_2 D\exp\{i\omega(\lambda'_2 z + x - ct)\} \end{aligned}\right\} \quad (4.12)$$

where C, D, E' are constants,

$$k'_i = \frac{B'_1\lambda'^2_i - \left(\rho' + \varepsilon_0\mu_0^2 H_0^2\right)c^2 + \left(A'_{11} + \mu_0 H_0^2\right)}{\lambda'_i\left(B'_2 + \mu_0 H_0^2\right)}, \quad i = 1, 2.$$

and

$$\lambda'^2_1 + \lambda'^2_2 = \frac{\begin{array}{c}\left(A'_{22} + \mu_0 H_0^2\right)\left\{\left(\rho' + \varepsilon_0\mu_0^2 H_0^2\right)c^2 - \left(A'_{11} + \mu_0 H_0^2\right)\right\} \\ +B'_1\left\{\left(\rho' + \varepsilon_0\mu_0^2 H_0^2\right)c^2 - B'_1\right\} + \left(B'_2 + \mu_0 H_0^2\right)^2\end{array}}{B'_1\left(A'_{22} + \mu_0 H_0^2\right)} \quad (4.12a)$$

$$\lambda'^2_1\lambda'^2_2 = \frac{\left\{\left(\rho' + \varepsilon_0\mu_0^2 H_0^2\right)c^2 - B'_1\right\}\left\{\left(\rho' + \varepsilon_0\mu_0^2 H_0^2\right)c^2 - \left(A'_{11} + \mu_0 H_0^2\right)\right\}}{B'_1\left(A'_{22} + \mu_0 H_0^2\right)}$$

$$\lambda'^2_3 = \frac{\rho' c^2 - B'_1}{B'_3} \quad (4.12b)$$

Applying boundary conditions, we get

$$A + B - C - D = 0 \quad (4.13)$$

$$E = E' \quad (4.14)$$

$$k_1 A + k_2 B - k'_1 C - k'_2 D = 0 \quad (4.15)$$

$$B_1(\lambda_1 - k_1)A + B_1(\lambda_2 - k_2)B + B'_1\left(\lambda'_1 + k'_1\right)C + B'_1\left(\lambda'_2 + k'_2\right)D = 0 \quad (4.16)$$

$$-B_3\lambda_3 E = B'_3\lambda'_3 E' \quad (4.17)$$

$$\begin{aligned} &(B_2 - B_1 - A_{22}\lambda_1 k_1)A + (B_2 - B_1 - A_{22}\lambda_2 k_2)B \\ &- \left(A'_{22}\lambda'_1 k'_1 - B'_2 + B'_1\right)C + \left(A'_{22}\lambda'_1 k'_1 - B'_2 + B'_1\right)D = 0 \end{aligned} \quad (4.18)$$

From equations (4.14) and (4.17), it follows that

$$E = E' = 0 \tag{4.19}$$

Thus, the conditions in (4.19) imply that there is no propagation of the transverse component of displacement.

Elimination of the indispensable constants A, B, C, D from (4.13), (4.15), (4.16) and (4.18) gives the following determinantal equation

$$\begin{vmatrix} 1 & 1 & -1 & -1 \\ k_1 & k_2 & -k_1' & -k_2' \\ B_1(\lambda_1 - k_1) & B_1(\lambda_2 - k_2) & B_1'(\lambda_1' + k_1') & B_1'(\lambda_2' + k_2') \\ B_2 - B_1 - A_{22}\lambda_1 k_1 & B_2 - B_1 - A_{22}\lambda_2 k_2 & A_{22}'\lambda_1' k_1' - B_2' + B_1' & A_{22}'\lambda_2' k_2' - B_2' + B_1' \end{vmatrix} = 0 \tag{4.20}$$

The equation (4.20) gives the wave velocity c of the general surface waves propagating along the common boundary of two fibre-reinforced electrically conducting elastic solid media under the influence of magnetic field. Since λ_1,λ_2,λ_1',λ_2',k_1,k_2,k_1',k_2' do not contain ω explicitly hence the wave velocity c obtained from (4.20) can not directly depend on ω which indicates that dispersion of the general wave form does not occur in the present case as we have seen in the classical case also.

Hence, neither fibre reinforcing nor the presence of magnetic field causes dispersion of general wave form.

5. Particular Cases

CaseI : Rayleigh wave: Rayleigh wave is a special case of the above general surface wave. In this case we consider a model where the medium M_1 is replaced by vacuum. Since the boundary $z = 0$ is adjacent to vacuum, it is free from surface traction. So the stress boundary condition in this case may be expressed as [20]

$$\sigma_{13} = 0, \quad \sigma_{33} = 0 \qquad \text{on} \quad z = 0.$$

Under such special circumstances, one obtains the following conditions from (4.16) and (4.18)

$$(\lambda_1 - k_1)\,A + (\lambda_2 - k_2)\,B = 0$$

$$(B_2 - B_1 - A_{22}\lambda_1 k_1)\,A + (B_2 - B_1 - A_{22}\lambda_2 k_2)\,B = 0$$

Eliminating the constants A and B we get the wave velocity equation for Rayleigh type of waves in the fibre-reinforced elastic medium under the influence of magnetic field as

$$\begin{vmatrix} \lambda_1 - k_1 & \lambda_2 - k_2 \\ B_2 - B_1 - A_{22}\lambda_1 k_1 & B_2 - B_1 - A_{22}\lambda_2 k_2 \end{vmatrix} = 0 \qquad (5.1)$$

For the sake of numerical calculation we take $\alpha + 2\mu_L = 2\mu_T,\ \beta = 0$. Many authors have made such type of assumption. Eringen [15] extended the concept of Poisson material in the case of non-local elastic solid. Acharya and Mandal [2] further extended the concept to non-local viscoelastic solid. Nowacki [19] adopted the concept of $\beta \to \infty$ in the problem of magnetoelasticity where β denotes the conduction of electricity. For convenience, we introduce the following

$$c_A^2 = \frac{\mu_0 H_0^2}{\rho},\ c_0^2 = \frac{1}{\varepsilon_0\mu_0},\ c_1^2 = \frac{\lambda + 2\mu_T}{\rho},\ c_2^2 = \frac{\mu_L}{\rho},\ s = \frac{c}{c_2}$$

$$\alpha_0 = 1 + \frac{c_A^2}{c_2^2} \times \frac{c_2^2}{c_0^2},\ \beta_0 = \frac{1 + \frac{c_A^2}{c_2^2} \times \frac{c_2^2}{c_0^2}}{1 + \frac{c_A^2}{c_2^2} \times \frac{c_2^2}{c_1^2}},\ \gamma_0 = \frac{c_2^2}{c_1^2},$$

and hence the equation (5.1) transforms to

$$\left(2 - \alpha_0 s^2\right)\left(2 - \beta_0 s^2\right) = 4\left(1 - \alpha_0 s^2\right)^{\frac{1}{2}}\left(1 - \beta_0\gamma_0 s^2\right)^{\frac{1}{2}} \qquad (5.2)$$

Case II: Love wave: Love wave is also a particular case of general surface wave. For the existence of Love waves one has to consider a layered semi-infinite medium in which the medium M_1 is an infinitely extended horizontal plate of finite thickness d and bounded by two horizontal plane surfaces $z = 0$ and $z = -d$. The medium M is semi-infinite as in the general case [fig.2].

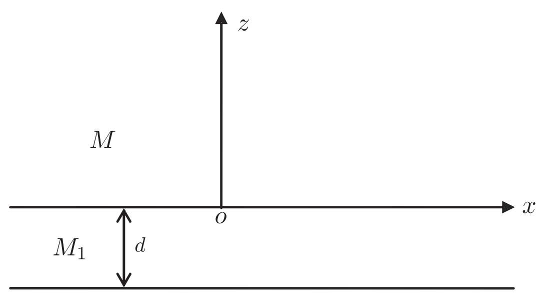

Figure 2. Geometry for the propagation of Love waves.

For the propagation of Love waves v is the only component of displacement vector to play the role and the boundary conditions are

(i) $v = v', \ \sigma_{23} = \sigma'_{23}$ at any point on $z = 0$,

(ii) $\sigma'_{23} = 0$ at $z = -d$.

Since v' satisfies the dynamical equation (2.13a) the solution valid for the medium M_1 of finite thickness d may be taken as

$$v' = E' \exp\left\{i\omega\left(\lambda'_3 z + x - ct\right)\right\} + F' \exp\left\{i\omega\left(-\lambda'_3 z + x - ct\right)\right\} \tag{5.3}$$

Since the medium M is a semi-infinite one, considering the regularity conditions satisfied by v as $z \to \infty$ the solution of (2.13) may be taken as

$$v = E \exp\left\{i\omega\left(-\lambda_3 z + x - ct\right)\right\} \tag{5.3a}$$

where λ_3 and λ'_3 are given by (4.9) and (4.12b) respectively.

Employing the boundary conditions, we get

$$\left.\begin{array}{l} E - E' - F' = 0, \\ -B_3\lambda_3 E - B'_3\lambda'_3 E' - B'_3\lambda'_3 F' = 0, \\ E' - e^{2i\omega\lambda_3 d} F' = 0. \end{array}\right\} \tag{5.4}$$

Eliminating E,E',F' one obtains

$$\begin{vmatrix} 1 & -1 & -1 \\ -B_3\lambda_3 & -B'_3\lambda'_3 & B'_3\lambda'_3 \\ 0 & 1 & -e^{2i\omega\lambda_3 d} \end{vmatrix} = 0 \tag{5.5}$$

which on simplification yields

$$\omega d = \frac{1}{\left\{c_T^2 \times \frac{\mu'_L}{\mu'_T} - \frac{\mu'_L}{\mu'_T}\right\}^{\frac{1}{2}}} \tan^{-1}\left[\frac{\mu_T}{\mu'_T}\left\{\frac{\frac{\mu_L}{\mu_T} - c_T^2 \times \frac{\mu'_L/\rho'}{\mu_T/\rho}}{c_T^2 \times \frac{\mu'_L}{\mu'_T} - \frac{\mu'_L}{\mu'_T}}\right\}^{\frac{1}{2}}\right] \tag{5.6}$$

in which $c_T = c\big/\sqrt{\mu'_L/\rho'}$.

The equation (5.6) gives the wave velocity of Love type of wave propagating in a fibre-reinforced model (fig.2) in presence of an initial magnetic field $(0, H_0, 0)$. It is interesting to note here that equation (5.6) is independent of any constraint due to the presence of initial magnetic field. Hence, the presence of the initial magnetic field can not modulate the Love wave velocity under consideration.

Case III: Stoneley waves: It is also a surface wave and may be considered as the generalized form of Rayleigh waves propagating at the common boundary of M and M_1. Hence, the wave velocity equation (4.20) for general surface waves may also be considered for the Stoneley waves in a fibre-reinforced magneto elastic media along the common boundary. Since the wave velocity equation (4.20) for Stoneley waves under the present circumstances does not contain ω explicitely, such types of waves are not dispersive like the classical one.

Special case I: In the absence of magnetic field ($H_0 = 0$) the wave velocity equation (4.20) transforms to

$$\begin{vmatrix} 1 & 1 & -1 & -1 \\ k_4 & k_5 & -k'_4 & -k'_5 \\ B_1(\lambda_4 - k_4) & B_1(\lambda_5 - k_5) & B'_1(\lambda'_4 + k'_4) & B'_1(\lambda'_5 + k'_5) \\ B_2 - B_1 - A_{22}\lambda_4 k_4 & B_2 - B_1 - A_{22}\lambda_5 k_5 & A'_{22}\lambda'_4 k'_4 - B'_2 + B'_1 & A'_{22}\lambda'_5 k'_5 - B'_2 + B'_1 \end{vmatrix} = 0 \tag{5.7}$$

where λ_4,λ_5, λ'_4,λ'_5, k_4, k_5, k'_4, k'_5 are given by the following

$$\lambda_4^2 + \lambda_5^2 = \frac{A_{22}(\rho c^2 - A_{11}) + B_1(\rho c^2 - B_1) + B_2^2}{B_1 A_{22}}, \quad \lambda_4^2 \lambda_5^2 = \frac{(\rho c^2 - B_1)(\rho c^2 - A_{11})}{B_1 A_{22}}$$

$$\lambda'^2_4 + \lambda'^2_5 = \frac{A'_{22}(\rho' c^2 - A'_{11}) + B'_1(\rho' c^2 - B'_1) + B'^2_2}{B'_1 A'_{22}}, \quad \lambda'^2_4 \lambda'^2_5 = \frac{(\rho' c^2 - B'_1)(\rho' c^2 - A'_{11})}{B'_1 A'_{22}}$$

and

$$k_i = \frac{B_1\lambda_i^2 - \rho c^2 + A_{11}}{\lambda_i B_2}, \quad k_i' = \frac{B_1'\lambda_i'^2 - \rho' c^2 + A_{11}'}{\lambda_i' B_2'}, \quad i = 4, 5.$$

The equation (5.7) represents the wave velocity equation for Stoneley waves propagated near the common boundary of two fibre-reinforced semi-infinite media.

Special case II: Stoneley wave velocity equation for a magnetoelastic media may be obtained from (4.20) (by taking $\mu_L = \mu_T = \mu,\ \alpha = 0,\ \beta = 0$ in which λ and μ are Lame elastic constants) as

$$\begin{vmatrix} 1 & 1 & -1 & -1 \\ k_1 & k_2 & -k_1' & -k_2' \\ B_1(\lambda_1 - k_1) & B_1(\lambda_2 - k_2) & B_1'(\lambda_1' + k_1') & B_1'(\lambda_2' + k_2') \\ B_2 - B_1 - A_{22}\lambda_1 k_1 & B_2 - B_1 - A_{22}\lambda_2 k_2 & A_{22}'\lambda_1' k_1' - B_2' + B_1' & A_{22}'\lambda_2' k_2' - B_2' + B_1' \end{vmatrix} = 0 \quad (5.8)$$

where $A_{11} = A_{22} = \lambda + 2\mu,\ \ B_1 = B_3 = \mu,\ \ B_2 = \lambda + \mu.$

Putting $H_0 = 0$ in (5.8), the classical Stoneley wave velocity equation may be obtained easily.

6. Numerical Results and Physical Discussions

Rayleigh waves:

Using commercially available software MS Excel and MathCAD 12, the values of Rayleigh wave velocity in non-dimensional form (c/c_2) has been calculated from (5.1) for different values of Alfven wave velocity parameter (c_A/c_2), conductivity parameter (c_2/c_0) and reinforcement parameter (c_2/c_1). The results have been depicted in graphs and are given in figure 3 and figure 4. Figure 3 gives the variation of Rayleigh wave velocity with respect to the Alfven wave velocity parameter for different values of electrical conductivity parameter $\left(\frac{c_2}{c_0} = 0, \frac{c_2}{c_0} = 0.7, \frac{c_2}{c_0} = 1, \frac{c_2}{c_0} = 1.3\right)$. The physical interpretation which may be made from figure 3 is that as the Alfven wave velocity increases phase velocity of Rayleigh wave decreases continuously in magnitude. Such decrements become more and more significant as the electrical conductivity increases. From the curve, corresponding to $\frac{c_2}{c_0} = 0$ it is observed that in the absence of electrical conductivity variations of Rayleigh wave velocity with respect to Alfven wave velocity is very small. For a particular value of Alfven wave velocity parameter, Rayleigh wave velocity decreases with the increase of electrical conductivity (c_2/c_0). Figure 4 gives the velocity of Rayleigh waves verses Alfven wave velocity for different type of fibre-reinforced material $(\frac{c_2}{c_1} = 0.463, \frac{c_2}{c_1} = 0.635, \frac{c_2}{c_1} = 0.825)$ when the electrical conductivity $\frac{c_2}{c_0} = 0.7$. In this case, the physical interpretation is that the phase velocity of Rayleigh wave decreases as the Alfven wave velocity increases. For a particular value of Alfven wave velocity, phase velocity decreases as the fibre-reinforcement parameter $\frac{c_2}{c_1}$ increases. The above discussion expresses the physical fact, in general, that the magnetic field as well as fibre-reinforcement which are generally neglected in corresponding classical problems, influences the propagation of Rayleigh waves to a considerable extent.

<u>Love waves</u>:

The frequency equation (5.6) expresses the fact that the phase velocity of Love wave depends upon frequency, densities of the media and fibre-reinforcing material parameters. It is also observed from this equation that the presence of initial magnetic field can not influence the propagation of Love waves in a model consisting of a fibre-reinforced layer and another

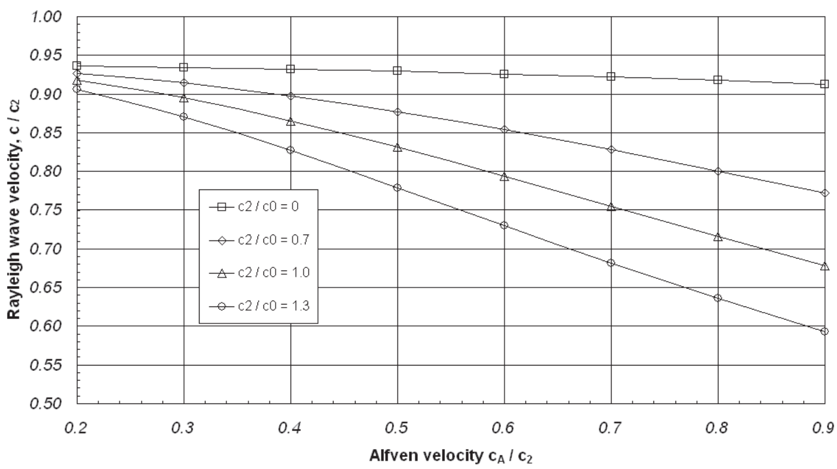

Figure 3. Rayleigh wave velocity (c/c_2) vs. Alfven velocity (c_A/c_2) for different conductivity (c_2/c_0) when the reinforcement parameter $c_2/c_1 = 0.463$.

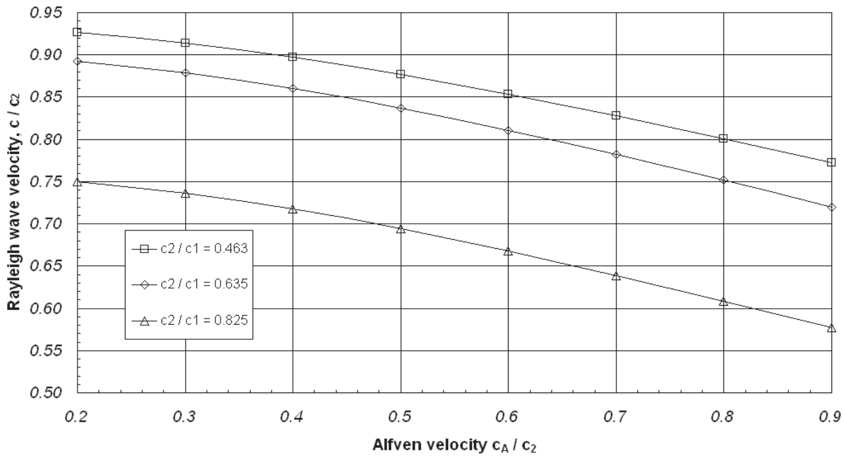

Figure 4. Rayleigh wave velocity (c/c_2) vs. Alfven velocity (c_A/c_2) for different fibre-reinforced material(c_2/c_1) when conductivity$c_2/c_0 = 0.7$.

elastic half-space of different fibre-reinforced materials. For the purpose of graphical representation, we take the values of material parameters and the densities of the media as

$$\mu_L = 5.66\times 10^9 N/m^2,\quad \mu_T = 2.46\times 10^9\, N/m^2,\quad \rho = 7800\, kg/m^3$$
$$\mu'_L = 2.45\times 10^9 N/m^2,\quad \mu'_T = 1.89\times 10^9\, N/m^2,\quad \rho' = 6500\, kg/m^3$$

Numerical values of $\frac{c}{\sqrt{\mu'_L/\rho'}}$ have been calculated from (5.6) for different values of ωd by using commercially available software MS Excel and Math-CAD 12. It is interesting to note from figure 5 that Love wave velocity propagated along the interface between two fibre-reinforced material media [fig.2], rapidly drops down to 1.05(approx) from its highest value 1.4(approx) as ωd increases from 0 to 3.5(approx). For values of ωd>3.5 the rate of decrement of Love wave velocity is very small under the present situation. The physical fact which emerges out of the above analysis is that fibre-reinforcement plays a vital role in the propagation of Love waves where as the presence of magnetic field can not influence the same. Moreover the thickness of the fibre-reinforced layer has a pronounced effect on the propagation of Love waves.

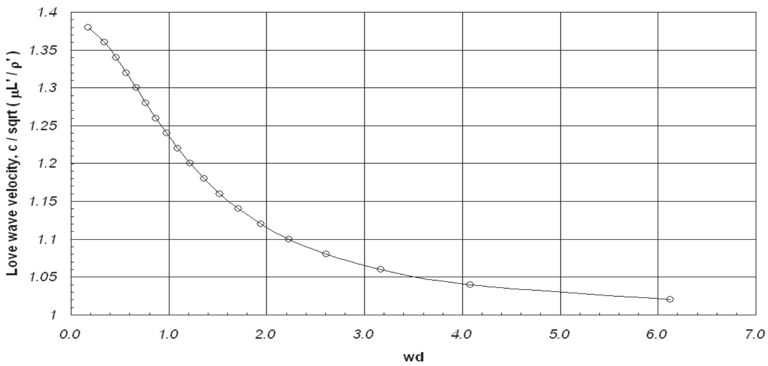

Figure 5. Love wave velocity $c/\sqrt{\mu'_L/\rho'}$ vs. ωd.

8. Conclusions

In the light of the above analysis, following conclusions may be made.

1. Fibre reinforcing modulates Rayleigh wave velocity to a considerable extent. Further modulation of wave velocity takes place due to the presence of initial magnetic field in a fibre-reinforced medium. Presence of initial magnetic field $(0, H_0, 0)$ in a fibre reinforcing material can not cause the dispersion of the general wave form for Rayleigh waves. Effect of electrical conductivity plays a significant role.
2. Initial magnetic field $(0, H_0, 0)$ can not influence Love wave propagation. Frequency (ω), thickness of the layer (d) and fibre reinforcing parameters have a salient influence on the Love wave propagation.
3. Numerical discussion of Stoneley wave is not pursued in this paper due to its complicated nature and cumbersome calculation. However from the wave velocity equation (4.20) we may state that the combined effect of initial magnetic field and fibre reinforcing has a significant role to play on the propagation of Stoneley waves.
4. Results obtained in this paper may be considered as more general in the sense that they include the combined effect of fibre reinforcing and initial magnetic field.

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