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SYMMETRICAL BROADWELL MODEL: CALCULATION OF SOLUTION

BY

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Broadwell equations, in symmetrical case, are equations (1), where $i = 1, 2, 3$. Unknowns functions $n_i(t)$ are densities, the variable t is the time. To simplify we omit coefficients, appearing in original Broadwell model. Then functions $n_i(t)$ are inverse of time , and they have same dimensions as $(1/t)$. We will search general solution of equations (1) on the form (2).

$$\frac{dn_i}{dt} + 3n_i^2 = \sum_{j=1}^{3} n_j^2, \tag{1}$$

$$n_i(t) = A(t) + \frac{\alpha(t)}{\varphi(t) + C_i}. \tag{2}$$

Functions $A(t)$, $\alpha(t)$ and $\varphi(t)$ are functions to be determined, instead quantities C_i are constants, also to be determined. Putting formulas (2) in equations (1), one obtains equations (3) :

$$\begin{aligned} &\dot{A} + 3A^2 \frac{\dot{\alpha} + 6A\alpha}{\varphi + C_i} + \frac{3\alpha^2 - \alpha\dot{\varphi}}{(\varphi + C_i)^2} \\ &= 3A^2 + 2A\alpha\Big\{\sum_{j=1}^{3} \frac{1}{\varphi + C_j}\Big\} + \alpha^2\Big\{\sum_{j=1}^{3} \frac{1}{(\varphi + C_j)^2}\Big\}. \end{aligned} \tag{3}$$

The right hand-side of equations (3) are independent of index i, instead left hand-side depends of that index. In order that equations (3) can be reduced to a single equation unique, it is sufficient to choose $\dot{\alpha} + 6A\alpha = 0$ and $3\alpha - \dot{\varphi} = 0$, what we will do. Equations (3) can be replaced by single

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equation (4), instead relations (2) become relations (5).

$$\frac{1}{6}\frac{d}{dt}\Big(\frac{\ddot{\varphi}}{\dot{\varphi}}\Big)=\frac{\ddot{\varphi}}{9}\Big\{\sum_{j=1}^{3}\frac{1}{\varphi+C_j}\Big\}-\frac{\varphi^2}{9}\Big\{\sum_{j=1}^{3}\frac{\dot{\varphi}}{\varphi+C_i}\Big\}, \tag{4}$$

$$n_i(t)=-\frac{\ddot{\varphi}}{6\dot{\varphi}}+\frac{1}{3}\frac{\dot{\varphi}}{\varphi+C_i}. \tag{5}$$

Equation (4), which replace system (1) can be integrated a first time under form (6), then a second time under form (7), from which we deduce that φ is inverse of a time, and that is the same for constants C_i.

$$\frac{\ddot{\varphi}}{\dot{\varphi}}=\frac{2}{3}\Big\{\sum_{j=1}^{3}\frac{\dot{\varphi}}{\varphi+C_j}\Big\}-K_1, \tag{6}$$

$$\log|\dot{\varphi}|=\frac{2}{3}\log|(\varphi+C_1)(\varphi+C_2)(\varphi+C_3)|-K_1t+K_2. \tag{7}$$

From equations (1) the sum of three densities $n_i(t)$ is a constant, we will note by $\frac{\overline{n}}{2}$. From equations (5) and (6), $\sum\limits_{i=1}^{6} n_i(t)=K_1=\overline{n}$. We will then put $\overline{n}t-K_2=\tau$, dimensionless variable, and $\exp(-\tau)=T$, also dimensionless, then $\overline{n}dt=d\tau=-\frac{dT}{T}$. We choose as initial time, the time t_0 corresponding to $\varphi=0$. Then we choose $K_2=\overline{n}t_0$, so that when $t=t_0$, $T=T_0=1$. From equation (7) we can successively write :

$$\frac{d\varphi}{dt}=\Big\{(\varphi+C_1)(\varphi+C_2)(\varphi+C_3)\Big\}^{\frac{2}{3}}T, \tag{8a}$$

$$T(\varphi)=1-\overline{n}\int_0^{\varphi}\frac{d\varphi}{\Big\{(\varphi+C_1)(\varphi+C_2)(\varphi+C_3)^{\frac{2}{3}}\Big\}}. \tag{8b}$$

Relation (8b) defines function $T(\varphi)$ and inverse $\varphi(T)$, consequently the three functions $\varphi(t)$, $\dot{\varphi}(t)$ and $\ddot{\varphi}(t)$. Then: $n_i=-\frac{1}{6}\frac{\ddot{\varphi}}{\dot{\varphi}}+\frac{1}{3}\frac{\dot{\varphi}}{\varphi+C_i}$, $\dot{\varphi}=\overline{n}\frac{d\varphi}{d\tau}$,

$$n_i = \frac{\overline{n}}{6}-\frac{\dot{\varphi}}{9}\Big\{\frac{1}{\varphi+C_1}+\frac{1}{\varphi+C_2}+\frac{1}{\varphi+C_3}\Big\}+\frac{1}{3}\frac{\dot{\varphi}}{\varphi+C_i}, \tag{9a}$$

$$\begin{aligned} n_i(\varphi) &= \frac{\overline{n}}{6}-\frac{T(\varphi)}{9}\Big\{\Big(\frac{1}{\varphi+C_1}+\frac{1}{\varphi+C_2}+\frac{1}{\varphi+C_3}\Big)-\frac{3}{\varphi+C_i}\Big\} \\ &\quad\times\Big\{(\varphi+C_1)(\varphi+C_2)(\varphi+C_3)\Big\}^{\frac{2}{3}}. \end{aligned} \tag{9b}$$

To summarize: solution of symmetrical Broadwell equations is defined, on a parametric form by relations (8b) et (9b).

Dimensions of different quantities. n_i being inverse of a time one can, from equations (1), multiply unknowns n_i by a constant k, and divide variable t by these constant. $A = -\ddot{\varphi}/(6\dot{\varphi})$ is also the inverse of a time, and is also multiplied by k. From formula (8a), φ is the inverse of a time, like n_i, and then like C_i, instead $\alpha = \dot{\varphi}/3$, is the square of the inverse of a time, like n_i^2. Then A, φ et C_i are multiplied by k, instead α is multiplied by k^2. Therefore one can choose the three constants C_i so that $C_1C_2C_3 = \overline{n}^3$, what we will do.

References

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