

EXISTENCE OF PERIODIC SOLUTIONS OF DISCRETE LOTKA-VOLTERRA SYSTEMS WITH DELAYS

BY

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Abstract. The existence of positive periodic solutions of discrete nonautonomous Lotka-Volterra systems with delays is studied by applying the continuation theorem of coincidence degree theory.

1. Introduction. Nonautonomous Lotka-Volterra systems with or without delays are very important mathematical models which describe multispecies population dynamics. The persistence, global asymptotic behaviors and existence of periodic solutions of these systems have been widely investigated by many authors [see 1-2]. Recently, Li [3] has studied the existence of periodic solutions for the following periodic Lotka-Volterra competition systems with distributed delays

$$(1.1) \quad \frac{du_i(t)}{dt} = u_i(t) \left[r_i(t) - a_{ii}(t)u_i(t) - \sum_{\substack{j=1 \\ j \neq i}}^n a_{ij} \int_{-T_{ij}}^0 K_{ij}(s)u_j(t+s)ds \right],$$
$$i = 1, 2, \dots, n.$$

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and the periodic state dependent delay Lotka-Volterra competition system

$$(1.2) \quad \frac{du_i(t)}{dt} = u_i(t) \left[r_i(t) - a_{ii}(t)u_i(t) - \sum_{\substack{j=1 \\ j \neq i}}^n a_{ij}(t)u_j(t - \tau_j(t, u_1(t), \dots, u_n(t))) \right], \quad i = 1, 2, \dots, n,$$

where $r_i, a_{ii} > 0, a_{ij} \geq 0 (j \neq i, i, j = 1, 2, \dots, n)$ are continuous ω -periodic functions, $T_{ij} \in (0, \infty) (j \neq i, i, j = 1, 2, \dots, n)$, $K_{ij} \in C([-T_{ij}, 0], (0, \infty))$, $\int_{-T_{ij}}^0 K_{ij}(s)ds = 1 (j \neq i, i, j = 1, 2, \dots, n)$, $\tau_i \in C(R^{n+1}, R)$ and $\tau_i, i = 1, 2, \dots, n$ are ω -periodic with respect to their first arguments, respectively.

Many authors [4–7] have argued that the discrete time models governed by difference equations are more appropriate than the continuous ones when the populations have non-overlapping generations, also, discrete time models can provide efficient computational models of continuous models for numerical simulations. It is reasonable to study discrete time Lotka-Volterra model governed by difference equations.

In this paper, we are concerned with the following discrete Lotka-Volterra systems with delays

$$(1.3) \quad u_i(p+1) = u_i(p) \exp \left\{ r_i(p) - a_{ii}(p)u_i(p) - \sum_{\substack{j=1 \\ j \neq i}}^n a_{ij}(p) \sum_{s=-T_{ij}}^0 k_{ij}(s)u_j(p+s) \right\}, \quad i = 1, 2, \dots, n$$

and

$$(1.4) \quad u_i(p+1) = u_i(p) \exp \left\{ r_i(p) - a_{ii}(p)u_i(p) - \sum_{\substack{j=1 \\ j \neq i}}^n a_{ij}(p)u_j(p - \tau_j(p, u_1(p), u_2(p), \dots, u_n(p))) \right\},$$

$$i = 1, 2, \dots, n,$$

which can be looked as discrete analogues of systems (1.1) and (1.2), respectively, where $r_i, a_{ii}, a_{ij} : Z \rightarrow R^+$, $i, j = 1, 2, \dots, n$, are all ω -periodic functions, ω is a fixed positive integer and $T_{ij}, j \neq i, i, j = 1, 2, \dots, n$ are positive integers, $k_{ij} : [-T_{ij}, 0] \rightarrow R^+$ satisfying $\sum_{s=-T_{ij}}^0 k_{ij}(s) = 1, j \neq i, i, j = 1, 2, \dots, n$, $\tau_j : R^{n+1} \rightarrow Z^+, j = 1, 2, \dots, n$ are ω -periodic with respect to their first arguments, respectively, where Z, Z^+, R^+ denote the sets of all integers, nonnegative integers and nonnegative real numbers, respectively, for $a \leq b \in Z$, we denote $[a, b] = \{a, a + 1, \dots, b\}$.

A very basic and important ecological problem associated with the study of multispecies population interaction in a periodic environment is the global existence of positive solution which plays the role played by the equilibrium of the autonomous models. To the best of our knowledge, no such work has been done for systems (1.3) and (1.4). Our purpose of this paper is by using Mawhin's continuous theorem [8] to study the global existence of positive periodic solutions of systems (1.3) and (1.4).

2. Existence of positive periodic solutions. In this section, based on the Mawhin's continuation theorem, we shall study the existence of at least one positive periodic solution of (1.1). First, we shall make some preparations.

Let X, Y be normed vector spaces, $L : \text{Dom } L \subset X \rightarrow Y$ be a linear mapping, and $N : X \rightarrow Y$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim \text{Ker } L = \text{codim Im } L < +\infty$ and $\text{Im } L$ is closed in Y . If L is a Fredholm mapping of index zero and there exist continuous projectors $P : X \rightarrow X$ and $Q : Y \rightarrow Y$ such that $\text{Im } P = \text{Ker } L, \text{Ker } Q = \text{Im } L = \text{Im } (I - Q)$, it follows that mapping $L|_{\text{Dom } L \cap \text{Ker } P} : (I - P)X \rightarrow \text{Im } L$ is invertible. We denote the inverse of that mapping by K_P . If Ω is an open bounded subset of X , the mapping N will be called L -compact on $\overline{\Omega}$ if $QN(\overline{\Omega})$ is bounded and $K_P(I - Q)N : \overline{\Omega} \rightarrow X$

is compact. Since $\text{Im } Q$ is isomorphic to $\text{Ker } L$, there exists an isomorphism $J: \text{Im } Q \rightarrow \text{Ker } L$.

Now, we introduce Mawhin's continuation theorem [8, p.40] as follows.

Lemma 2.1. *Let $\Omega \subset X$ be an open bounded set and let $N: X \rightarrow Y$ be a continuous operator which is L -compact on $\bar{\Omega}$. Assume*

- (a) *for each $\lambda \in (0, 1)$, $x \in \partial\Omega \cap \text{Dom } L$, $Lx \neq \lambda Nx$,*
- (b) *for each $x \in \partial\Omega \cap \text{Ker } L$, $QNx \neq 0$,*
- (c) *$\deg(JNQ, \Omega \cap \text{Ker } L, 0) \neq 0$.*

Then $Lx = Nx$ has at least one solution in $\bar{\Omega} \cap \text{Dom } L$.

Lemma 2.2. ([9]) *Let $f: Z \rightarrow R$ be ω periodic, i.e., $f(k + \omega) = f(k)$, then for any fixed $k_1, k_2 \in [0, \omega - 1]$ and any $k \in Z$, one has*

$$\begin{aligned} f(k) &\leq f(k_1) + \sum_{s=0}^{\omega-1} |f(s+1) - f(s)|, \\ f(k) &\geq f(k_2) - \sum_{s=0}^{\omega-1} |f(s+1) - f(s)|. \end{aligned}$$

In what follows, we shall use the notation

$$\bar{f} = \frac{1}{\omega} \sum_{k=0}^{\omega-1} f(k),$$

where f is an ω -periodic function.

Theorem 2.1. *Assume that the system of equations*

$$(2.1) \quad \sum_{j=1}^n \bar{a}_{ij} \exp\{u_j\} = \bar{r}_i, \quad i = 1, 2, \dots, n$$

has a unique solution $(u_1^*, u_2^*, \dots, u_n^*)^T \in R^n$. If

$$\bar{r}_i - \sum_{\substack{j=1 \\ j \neq i}}^n \bar{a}_{ij} \exp\{M_j\} > 0, \quad i = 1, \dots, n,$$

where

$$M_i = \ln \frac{\bar{r}_i}{\bar{a}_{ii}} + 2\bar{r}_i\omega, \quad i = 1, \dots, n,$$

then system (1.3) has at least one positive ω -periodic solution.

Proof. Consider the following system

$$(2.2) \quad \begin{aligned} x_i(p+1) - x_i(p) &= r_i(p) - a_{ii}(p) \exp\{x_i(p)\} \\ &- \sum_{\substack{j=1 \\ j \neq i}}^n a_{ij}(p) \sum_{s=-T_{ij}}^0 k_{ij}(s) \exp\{x_i(p+s)\}, \quad i = 1, \dots, n, \end{aligned}$$

where $r_i, a_{ij}, T_{ij}, k_{ij} (i, j = 1, 2, \dots, n)$ are the same as those in system (1.3).

We can easily see that if system (2.2) has an ω -periodic solution $(x_1^*(t), \dots, x_n^*(t))^T$, then $(u_1^*(t), \dots, u_n^*(t))^T = (\exp\{x_1^*(t)\}, \dots, \exp\{x_n^*(t)\})^T$ is a positive ω -periodic solution of system (1.3), so we only need to show that system (2.2) has at least one ω -periodic solution in order to complete the proof. To apply the continuation theorem of coincidence degree theory to establish the existence of an ω -periodic solution of (2.2), we take

$$\begin{aligned} X = Y = \{y(k)\} &= \{y = (y_1(k), \dots, y_n(k)) \in R^n, y_i(k + \omega) = y_i(k), k \in Z, \\ & \quad i = 1, \dots, n\} \end{aligned}$$

and denote

$$\|y\| = \sum_{i=1}^n |y_i|_0, \quad |y|_0 = \max_{k \in [0, \omega-1]} |y(k)|,$$

then X is a Banach space. Let

$$X_0 = \left\{ y = \{y(k)\} \in X, \sum_{k=0}^{\omega-1} y(k) = 0 \right\},$$

$$X_c = \{y = \{y(k)\} \in X, y(k) = h \in R^n, k \in Z\},$$

then it is easy to check that X_0 and X_c are both closed linear subspace of X and

$$X = X_0 \oplus X_c, \quad \dim X_c = n.$$

Set

$$(Lx)(p) = x(p+1) - x(p), \quad p \in Z, \quad x \in X,$$

and $N : X \rightarrow X$,

$$(Nx)(p) = \begin{pmatrix} r_1(p) - a_{11}(p) \exp\{x_1(p)\} - \sum_{j=2}^n a_{1j}(p) \sum_{s=-T_{1j}}^0 k_{1j}(s) \exp\{x_j(p+s)\} \\ \vdots \\ r_n(p) - a_{nn}(p) \exp\{x_n(p)\} - \sum_{j=1}^{n-1} a_{nj}(p) \sum_{s=-T_{nj}}^0 k_{nj}(s) \exp\{x_j(p+s)\} \end{pmatrix},$$

$p \in Z, x \in X,$

then it is trivial to see that L is a bounded linear operator with

$$\text{Ker } L = X_c, \quad \text{Im } L = X_0, \quad \dim \text{Ker } L = \text{codim Im } L = n,$$

hence, L is a Fredholm mapping of index zero.

Define two projectors P and Q as

$$Qy = Py = \frac{1}{\omega} \sum_{s=0}^{\omega-1} y(s), \quad y \in X.$$

It is not difficult to show that P and Q are continuous projectors such that

$$\text{Im } P = \text{Ker } L \text{ and } \text{Im } L = \text{Ker } Q = \text{Im } (I - Q).$$

Furthermore, the generalized inverse (to L) $K_P : \text{Im } L \rightarrow \text{Ker } P \cap \text{Dom } L$ exists, which has the form

$$(K_P y)(n) = \sum_{i=0}^{n-1} y(i) - \frac{1}{\omega} \sum_{i=1}^{\omega} \sum_{s=0}^{i-1} y(s).$$

Obviously, QN and $K_P(I - Q)N$ are continuous. Since X is a finite-dimensional Banach space, one can easily show that $\overline{K_P(I - Q)N(\bar{\Omega})}$ is compact for any open bounded set $\Omega \subset X$. Moreover, $QN(\bar{\Omega})$ is bounded, and hence N is L -compact on $\bar{\Omega}$ with any open bounded set $\Omega \subset X$.

Now we are in a position to search for an appropriate open, bounded subset $\Omega \subseteq X$ for the application of the continuation theorem. Corresponding to the operator equation $Lx = \lambda Nx, \lambda \in (0, 1)$, we have

$$(2.3) \quad \begin{aligned} x_i(p+1) - x_i(p) &= \lambda r_i(p) - \lambda a_{ii}(p) \exp\{x_i(p)\} \\ &- \lambda \sum_{\substack{j=1 \\ j \neq i}}^n a_{ij}(p) \sum_{s=-T_{ij}}^0 k_{ij}(s) \exp\{x_j(p+s)\}, \quad i = 1, \dots, n. \end{aligned}$$

Suppose that $x = \{x(p)\} = \{(x_1(p), x_2(p), \dots, x_n(p))\} \in X$ is a solution of (2.3) for some $\lambda \in (0, 1)$. Summing on both sides of (2.3) from 0 to $\omega - 1$ with respect to p produces

$$(2.4) \quad \begin{aligned} \omega \bar{r}_i &= \sum_{p=0}^{\omega-1} a_{ii}(p) \exp\{x_i(p)\} \\ &+ \sum_{p=0}^{\omega-1} \sum_{\substack{j=1 \\ j \neq i}}^n a_{ij}(p) \sum_{s=-T_{ij}}^0 k_{ij}(s) \exp\{x_j(p+s)\}, \quad i = 1, \dots, n. \end{aligned}$$

Let

$$x_i(\xi_i) = \max\{x_i(k), k \in [0, \omega - 1]\}, i = 1, 2, \dots, n$$

and

$$x_i(\eta_i) = \min\{x_i(k), k \in [0, \omega - 1]\}, i = 1, 2, \dots, n,$$

thus

$$\begin{aligned} & \omega \bar{r}_i - \sum_{p=0}^{\omega-1} a_{ii}(p) \exp\{x_i(\eta_i)\} \\ & \geq \omega \bar{r}_i - \sum_{p=0}^{\omega-1} a_{ii}(p) \exp\{x_i(p)\} \\ & = \sum_{p=0}^{\omega-1} \sum_{\substack{j=1 \\ j \neq i}}^n a_{ij}(p) \sum_{s=-T_{ij}}^0 k_{ij}(s) \exp\{x_j(p+s)\} \geq 0, \quad i = 1, \dots, n, \end{aligned}$$

that is,

$$(2.5) \quad \exp\{x_i(\eta_i)\} \leq \frac{\bar{r}_i}{\bar{a}_{ii}}, \quad \text{or,} \quad x_i(\eta_i) \leq \ln \frac{\bar{r}_i}{\bar{a}_{ii}}, \quad i = 1, \dots, n.$$

Furthermore, from (2.3), we can get

$$\begin{aligned} (2.6) \quad \sum_{p=0}^{\omega-1} |x_i(p+1) - x_i(p)| & \leq \lambda \left\{ \sum_{p=0}^{\omega-1} r_i(p) + \sum_{p=0}^{\omega-1} a_{ii}(p) \exp\{x_i(p)\} \right. \\ & \quad \left. + \sum_{\substack{j=1 \\ j \neq i}}^n a_{ij}(p) \sum_{s=-T_{ij}}^0 k_{ij}(s) \exp\{x_j(p+s)\} \right\} \\ & < 2\bar{r}_i \omega, \quad i = 1, \dots, n. \end{aligned}$$

From (2.5)–(2.6) and Lemma 2.2, it follows that for any $p \in Z$, we have

$$\begin{aligned} (2.7) \quad x_i(p) & \leq x_i(\eta_i) + \sum_{s=0}^{\omega-1} |x_i(s+1) - x_i(s)| \\ & \leq \ln \frac{\bar{r}_i}{\bar{a}_{ii}} + 2\bar{r}_i \omega := M_i, \quad i = 1, \dots, n. \end{aligned}$$

Similarly, from (2.4), we obtain

$$\omega \bar{r}_i \leq \sum_{p=0}^{\omega-1} a_{ii}(p) \exp\{x_i(\xi_i)\} + \sum_{p=0}^{\omega-1} \sum_{\substack{j=1 \\ j \neq i}}^n a_{ij}(p) \sum_{s=-T_{ij}}^0 k_{ij}(s) \exp\{x_j(p+s)\},$$

$$i = 1, \dots, n.$$

that is,

$$\begin{aligned} \exp\{x_i(\xi_i)\} \omega \bar{a}_{ii} &\geq \omega \bar{r}_i - \sum_{p=0}^{\omega-1} \sum_{\substack{j=1 \\ j \neq i}}^n a_{ij}(p) \sum_{s=-T_{ij}}^0 k_{ij}(s) \exp\{x_j(p+s)\} \\ &\geq \omega \bar{r}_i - \sum_{p=0}^{\omega-1} \sum_{\substack{j=1 \\ j \neq i}}^n a_{ij}(p) \exp\{M_j\} \\ &= \omega \bar{r}_i - \omega \sum_{\substack{j=1 \\ j \neq i}}^n \bar{a}_{ij} \exp\{M_j\}, \quad i = 1, \dots, n. \end{aligned}$$

Therefore, we get

$$(2.8) \quad x_i(\xi_i) \geq \ln \left\{ \frac{\bar{r}_i - \sum_{\substack{j=1 \\ j \neq i}}^n \bar{a}_{ij} \exp\{M_j\}}{\bar{a}_{ii}} \right\}, \quad i = 1, \dots, n.$$

From (2.6), (2.8) and Lemma 2.2, it follows that for any $p \in Z$, we have

$$(2.9) \quad \begin{aligned} x_i(p) &\geq x_i(\xi_i) - \sum_{s=0}^{\omega-1} |x_i(s+1) - x_i(s)| \\ &\geq \ln \left\{ \frac{\bar{r}_i - \sum_{\substack{j=1 \\ j \neq i}}^n \bar{a}_{ij} \exp\{M_j\}}{\bar{a}_{ii}} \right\} - 2\bar{r}_i \omega := m_i, \quad i = 1, \dots, n. \end{aligned}$$

From (2.7) and (2.9), it follows that

$$|x_i(p)| \leq \max\{|M_i|, |m_i|\} := \bar{M}_i, \quad p \in Z, \quad i = 1, 2, \dots, n.$$

Clearly, \bar{M}_i are independent of λ . Denote $M = \sum_{i=1}^n M_i + M_0$, M_0 is taken sufficiently large such that the unique solution $(u_1^*, u_2^*, \dots, u_n^*)^T$ of system (2.1) satisfies $\|(u_1^*, u_2^*, \dots, u_n^*)^T\| < M$. Now, we take $\Omega = \{x \in X, \|x\| < M\}$. This Ω satisfies condition (a) in Lemma 2.1.

When $x \in \partial\Omega \cap \text{Ker}L = \partial\Omega \cap R^n$, x is a constant vector in R^n with $\|x\| = M$. Then

$$QNx = \begin{pmatrix} \bar{r}_1 - \sum_{j=1}^n \bar{a}_{1j} \exp\{x_j\} \\ \vdots \\ \bar{r}_n - \sum_{j=1}^n \bar{a}_{nj} \exp\{x_j\} \end{pmatrix} < 0, \quad i = 1, 2, \dots, n.$$

Furthermore, take $J = I : \text{Im}Q \rightarrow \text{Ker}L, (x_1, x_2, \dots, x_n)^T \mapsto (x_1, x_2, \dots, x_n)^T$. In view of the assumption that (2.1) has a unique solution $(u_1^*, u_2^*, \dots, u_n^*)^T \in R^n$, by a straightforward computation, we have

$$\deg(JQN, \Omega \cap \text{Ker}L, 0) = \text{sgn}\{(-1)^n [\det(\bar{a}_{ij})] e^{\sum_{i=1}^n u_i^*}\} \neq 0.$$

Thus, by Lemma 2.1, we conclude that $Lx = Nx$ has at least one solution in X , that is, (2.1) has at least one ω -periodic solution. Therefore, (1.3) has at least one positive ω -periodic solution. The proof is complete.

Similar to the proof of Theorem 2.1, one can show that

Theorem 2.2. *Assume that the system of equations*

$$\sum_{j=1}^n \bar{a}_{ij} \exp\{u_j\} = \bar{r}_i, \quad i = 1, 2, \dots, n$$

has a unique solution $(u_1^*, u_2^*, \dots, u_n^*)^T \in R^n$. If

$$\bar{r}_i - \sum_{\substack{j=1 \\ j \neq i}}^n \bar{a}_{ij} \exp\{M_j\} > 0, \quad i = 1, \dots, n,$$

where

$$M_i = \ln \frac{\bar{r}_i}{\bar{a}_{ii}} + 2\bar{r}_i\omega, \quad i = 1, \dots, n,$$

then system (1.4) has at least one positive ω -periodic solution.

3. Examples. Consider the systems

$$(3.1) \quad \begin{cases} u_1(p+1) = u_1(p) \exp \left\{ 0.2 - (20 - \cos p\pi)u_1(p) \right. \\ \qquad \qquad \qquad \left. - (2 - \cos p\pi) \sum_{s=-T_{12}}^0 k_{12}(s)u_2(p+s) \right\}, \\ u_2(p+1) = u_2(p) \exp \left\{ 0.3 - 0.2 \cos p\pi - 10u_2(p) \right. \\ \qquad \qquad \qquad \left. - (4 - 3 \cos 3p\pi) \sum_{s=-T_{21}}^0 k_{21}(s)u_1(p+s) \right\} \end{cases}$$

and

$$(3.2) \quad \begin{cases} u_1(p+1) = u_1(p) \exp \{ 0.2 - (20 - \cos p\pi)u_1(p) \\ \qquad \qquad \qquad - (2 - \cos p\pi)u_2(p - \tau_2(p, u_1(p), u_2(p))) \}, \\ u_2(p+1) = u_2(p) \exp \{ 0.3 - 0.2 \cos p\pi - 10u_2(p) \\ \qquad \qquad \qquad - (4 - 3 \cos 3p\pi)u_1(p - \tau_1(p, u_1(p), u_2(p))) \}, \end{cases}$$

where $T_{ij}, j \neq i, i, j = 1, 2$ are positive integers, $k_{ij} : [-T_{ij}, 0] \rightarrow R^+$ satisfying $\sum_{s=-T_{ij}}^0 k_{ij}(s) = 1, j \neq i, i, j = 1, 2$, $\tau_j : R^2 \rightarrow Z^+, j = 1, 2$ are 2-periodic with respect to their first arguments, respectively. It is easy to see that $\bar{r}_1 = 0.2, \bar{r}_2 = 0.3, \bar{a}_{11} = 20, \bar{a}_{12} = 2, \bar{a}_{21} = 4, \bar{a}_{22} = 10$. Hence, one can easily check that the system

$$\sum_{j=1}^2 \bar{a}_{ij} \exp\{u_j\} = \bar{r}_i, \quad i = 1, 2$$

has a unique solution $(u_1^*, u_2^*)^T = (\ln \frac{7}{960}, \ln \frac{13}{480})^T \in R^2$ and

$$\bar{r}_1 - \bar{a}_{12} \exp \left\{ \ln \frac{\bar{r}_2}{\bar{a}_{22}} + 4\bar{r}_2 \right\} = 0.2 - 2 \exp \left\{ \ln \frac{0.3}{10} + 4 \times 0.3 \right\} > 0,$$

$$\bar{r}_2 - \bar{a}_{21} \exp \left\{ \ln \frac{\bar{r}_1}{a_{11}} + 4\bar{r}_1 \right\} = 0.3 - 4 \exp \left\{ \ln \frac{0.2}{20} + 4 \times 0.2 \right\} > 0.$$

Therefore, according to Theorem 2.1 and Theorem 2.2, systems (3.1) and (3.2) have at least one positive 2-periodic solution, respectively.

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