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HYDROMAGNETIC FREE CONVECTION AND MASS TRANSFER FLOW WITH JOULE HEATING, THERMAL DIFFUSION, HEAT SOURCE AND HALL CURRENT

BY

ATUL KUMAR SINGH, AJAY KUMAR SINGH AND N. P. SINGH

Abstract. The present paper deals with the combined effect of viscous dissipation, Joule heating, transpiration, heat source, thermal diffusion and Hall current on the hydromagnetic free convection and mass transfer flow of an electrically conducting, viscous, homogeneous, incompressible fluid past an infinite vertical porous plate. The governing partial differential equations of the hydromagnetic free convective boundary layer flow are reduced to non-linear ordinary differential equations by introducing suitable similarity transformations and are solved analytically. Solutions for the velocity field, the temperature field and the concentration field are obtained and discussed with the help of graphs. In addition, expressions for the skin-friction, the heat transfer and the mass transfer are also derived and are discussed with the help of tables.

1. Introduction. Hall currents effects are likely to be important in many astrophysical and geophysical situations as well as in engineering problems such as Hall accelerators, constructions of turbines and centrifugal machines. At present the study of magnetohydrodynamic flows has been motivated by several important problems such as MHD generators, applications in induction flow meters, maintenance of secular variations of the

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earth magnetic field, the internal rotation rate of sun, the structure of rotating magnetic stars, the planetary and solar dynamic problems, etc.

A comprehensive discussion of Hall effects is given by Cowling (1957). The boundary layer flow of electrically conducting, viscous, incompressible fluid with Hall effects has been studied by Pop (1971), Gupta (1975), Dutta and Jana (1976), Singh et al. (2000), Singh (2001) and several authors under various boundaries. Comprehensive studies have been carried out on hydromagnetic free convection and mass transfer flow with Hall currents by many workers, some of them are Dutta and Mazumdar (1976), Raptis and Kafoussias (1982), Raptis and Ram (1984) and Sattar (1994), Singh et al. (1999) and Singh et al. (2000).

More recently, Alam and Sattar (2000) have studied unsteady MHD free convection and mass transfer flow with Hall current, viscous dissipation and Joule heating. In this study the heat source effect and the thermal diffusion effect have been neglected where as in a convective fluid flow when the flow is caused by temperature and concentration differences, the heat source effect and the thermal diffusion effect can not be ignored [Eckert and Drake (1972)]. In view of the importance of the thermal diffusion effect, Jha and Singh (1990) and Jha (1991) studied the free convection and mass transfer flow of a viscous fluid past an infinite vertical plate taking into account the heat source effect and the thermal diffusion effect respectively. However, heat and mass transfer flow taking Hall current, viscous dissipation, Joule heating, heat source and thermal diffusion effect into account has not been studied. Therefore it is proposed to study of the steady hydromagnetic free convection and mass transfer flow with Hall current, viscous dissipation, Joule heating, transpiration, heat source and thermal diffusion effects. The similarity solutions of the governing equations are obtained for large suction by employing the perturbation technique demonstrated by Bestman (1990).

2. Formulation of the problem. Consider the steady two dimensional free convection and mass transfer flow of an incompressible, electrically conducting, viscous fluid past an infinite, vertical, porous plate in presence of transverse magnetic field taking into account the Hall current, the viscous dissipation, the Joule heating and the thermal dissipation. We assume a cartesian coordinate system taking x-axis along the plate in the upward direction and a straight line perpendicular to it as y-axis. The flow is assumed to be in x direction. The plate is assumed to be suddenly accelerated in its own plane with a uniform velocity U_0. The plate temperature and concentration are quickly raised from T_∞ to T_w and C_∞ to C_w respectively. Let $\vec{q}(u, v, w)$ be the velocity component of the fluid along the axes. Using the relation $\vec{\nabla} \cdot \vec{H} = 0$ for the magnetic field $\vec{H} = (H_x, H_y, H_z)$, we obtain $H_y = H_0$ so that $\vec{H} = (0, H_0, 0)$. Using the relation $\vec{\nabla} \cdot \vec{J} = 0$ for current density $\vec{J} = (J_x, J_y, J_z)$ we obtain $J_y =$ constant. Since the plate is non conducting $J_y = 0$ at the plate and hence zero everywhere. Further the plate is infinite in extent, all physical quantities dependent on x and y only. Also we consider that the accelerated plate is with constant plate temperature and the suction velocity is taken to be a function of x at the plate as $v_w(x)$. In addition, Joule heating term is added in the energy equation and thermal diffusion term is added in concentration equation.

Taking Hall current into account the generalized Ohm's law in absence of electric field, the electron pressure (for a partially or ionised gas), the thermo electric pressure and the ion slip are negligible [Cowling (1957) and Mayer (1958)]. Hence, we have

(2.1) $$J_x = \frac{\sigma \mu_e H_0}{1+m^2}(mu - w)$$

(2.2) $$J_y = \frac{\sigma \mu_e H_0}{1+m^2}(u + mw)$$

where σ, μ_e, H_0 are, respectively, the electric conductivity, the magnetic permeability, the magnetic intensity and $m = w_e\tau_e$ is the Hall parameter.

Under the above stated assumptions and usual Boussinesq's approximation, the governing equations relevant to the problem are:

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{2.3}$$

Momentum equations:

$$\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= \vartheta\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) \\ &\quad -\frac{\sigma\mu_e^2 H_0^2(x)}{\rho(1+m^2)}(u + mw) \end{aligned} \tag{2.4}$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} = \vartheta\frac{\partial^2 w}{\partial y^2} + \frac{\sigma\mu_e^2 H_0^2(x)}{\rho(1+m^2)}(mu - w). \tag{2.5}$$

Energy equation:

$$\begin{aligned} \frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} &= \frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\vartheta}{C_p}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] \\ &\quad + \left(\frac{\sigma\mu_e^2 H_0^2}{\rho(1+m^2)}\right)\frac{(u^2+w^2)}{C_p} + Q_0(T_\infty - T). \end{aligned} \tag{2.6}$$

Concentration Equation:

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_M\frac{\partial^2 C}{\partial y^2} + D_T\frac{\partial^2 T}{\partial y^2}. \tag{2.7}$$

The boundary conditions are:

$$\begin{aligned} &u = U_0, \quad v = v_w(x), \quad w = U_0, \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0 \\ &u \to 0, \quad w \to 0, \quad T \to T_\infty, \quad C \to C_\infty \qquad\qquad \text{as} \quad y \to \infty \end{aligned} \tag{2.8}$$

where u, v, w, are the components of velocity field, U_0 the free stream velocity, T the temperature of the flow field, T_w the temperature at the plate, T_∞ the temperature of the fluid at infinity, H_0 the magnetic field intensity, C the species concentration, C_w the concentration at the porous plate, C_∞ the species concentration at infinity, C_p the specific heat at constant pressure, D_M the molecular diffusivity, D_T the thermal diffusivity, σ the electrical conductivity, μ_e the magnetic permeability, β the coefficient of volume expansion, β^* the coefficient of volume expansion with concentration and the other symbols have their usual meaning.

3. Solution of the problem. To solve the partial differential equations (2.4) - (2.7), we adopt the usual similarity technique. For reasons of similarity, we introduce the following local similarity variables:

$$\Psi = \sqrt{2\vartheta x U_0} f(\eta), \quad \eta = y\sqrt{\frac{U_0}{2\vartheta x}}, \quad f'(\eta) = \frac{u}{U_0}, \quad g'(\eta) = \frac{w}{U_0},$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad \text{and} \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}.$$

From continuity equation (2.3), we have:

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x}. \tag{3.1}$$

Introducing the above mentioned non-dimensional quantities of usual similarity technique and integrating both sides of the equation (3.1), we obtain:

$$v = \sqrt{\frac{\vartheta U_0}{2x}}(\eta f' - f) \quad \text{and} \quad u = U_0 f'(\eta). \tag{3.2}$$

Introducing the above stated non-dimensional quantities, u and v from

(3.2) in equations (2.4) - (2.7), we obtain:

$$f''' + ff'' + G_r\theta + C_m\phi - \frac{M}{1+m^2}(f' + mg') = 0 \tag{3.3}$$

$$g''' + fg'' + \frac{M}{1+m^2}(mf' - g') = 0 \tag{3.4}$$

$$\theta'' + P_r f\theta' - SP_r\theta + J_h P_r \left[(f''^2 + g''^2) + \frac{M}{1+m^2}(f'^2 + g'^2)\right] = 0 \tag{3.5}$$

$$\phi'' + S_c f\phi' + S_0 S_c \theta'' = 0 \tag{3.6}$$

where $P_r = \frac{\mu C_p}{K}$ (Prandtl number),
$S_c = \frac{\vartheta}{D_M}$ (Schmidt number),
$S_0 = \left(\frac{T_w - T_\infty}{C_w - C_\infty}\right)\frac{D_T}{\vartheta}$ (Soret number),
$M = \frac{\sigma' \mu_e^2 H_0^2(x) 2x}{\rho U_0}$ (Magnetic parameter),
$G_r = \frac{g\beta(T_w - T_\infty)2x}{U_0^2}$ (Grashof number),
$S = \frac{2Q_0 x}{U_0}$ (Heat source parameter),
$G_m = \frac{g\beta^*(C_w - C_\infty)2x}{U_0^2}$ (Modified Grashof number),
$J_h = \frac{U_0^2}{C_p(T_w - T_\infty)}$ (Current density).

The boundary conditions (2.8) are transformed to:

$$\begin{aligned} &f = f_w, \quad f' = 1, \quad g = f_w, \quad g' = 1, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad \eta = 0 \\ &f' = 0, \quad g' = 0, \quad \theta = 0, \quad \phi = 0 \quad \text{as} \quad \eta \to \infty \end{aligned} \tag{3.7}$$

where $f_w = -v_0(x)\sqrt{\frac{2x}{\vartheta U_0}}$ and primes denotes the derivatives with respect to η. Here, $f_w > 0$ denotes the injection and $f_w < 0$ the suction.

To obtain solution for large suction, we make the following transformations:

$$\begin{aligned} &\xi = \eta f_w, \quad f(\eta) = f_w F(\xi), \quad g(\eta) = f_w E(\xi), \\ &\theta(\eta) = f_w^2 G(\xi), \quad \phi(\eta) = f_w^2 H(\xi). \end{aligned} \tag{3.8}$$

Using (3.8), in (3.3)-(3.6), we obtain:

(3.9) $F''' + FF'' - \varepsilon\left[\frac{M}{1+m^2}(F' + mE') - (G_rG + G_mH)\right] = 0$

(3.10) $E''' + FE'' + \varepsilon\left[\frac{M}{1+m^2}(mF' - E')\right] = 0$

(3.11) $G'' + P_rFG' - \varepsilon SP_rG + J_hP_r\left[\frac{M}{1+m^2}(F'^2 + E'^2) + \frac{1}{\varepsilon}(F''^2 + E''^2)\right] = 0$

(3.12) $H'' + S_cFH' + S_cS_0G'' = 0.$

The boundary conditions (3.7) reduce to:

(3.13)
$$\begin{aligned} &F(0) = 1, \quad F'(0) = \varepsilon, \quad F'(\infty) = 1, \quad E(0) = 1, \quad E'(0) = \varepsilon, \\ &E'(\infty) = 0, \quad G(0) = \varepsilon, \quad G(\infty) = 0, \quad H(0) = \varepsilon, \quad H(\infty) = 0, \end{aligned}$$

where $\varepsilon = \left(\frac{1}{f_w^2}\right)$ is very small as for large suction $f_w > 1$. Hence, we can expand F, E, G and H in terms of ε as follows:

(3.14) $$F(\xi) = 1 + \varepsilon F_1(\xi) + \varepsilon^2 F_2(\xi) + \cdots$$

(3.15) $$E(\xi) = 1 + \varepsilon E_1(\xi) + \varepsilon^2 E_2(\xi) + \cdots$$

(3.16) $$G(\xi) = \varepsilon G_1(\xi) + \varepsilon^2 G_2(\xi) + \cdots$$

(3.17) $$H(\xi) = \varepsilon H_1(\xi) + \varepsilon^2 H_2(\xi) + \cdots$$

Introducing $F(\xi)$, $E(\xi)$, $G(\xi)$ and $H(\xi)$ in equations (3.9) - (3.12) and considering up to order $O(\varepsilon^2)$, we get following two sets of ordinary differential equations and corresponding boundary conditions:

First order $O(\varepsilon)$:

(3.18) $F_1''' + F_1'' = 0$

$$E_1''' + E_1'' = 0 \tag{3.19}$$

$$G_1'' + P_r G_1' + J_h P_r (F_1''^2 + E_1''^2) = 0 \tag{3.20}$$

$$H_1'' + S_c H_1' + S_c S_0 G_1'' = 0 \tag{3.21}$$

$$\begin{aligned} &F_1(0) = 0, \quad F_1'(0) = 1, \quad F_1'(\infty) = 0, \quad E_1(0) = 0, \quad E_1'(0) = 1, \\ &E_1'(\infty) = 0, \quad G_1(0) = 1, \quad G_1(\infty) = 0, \quad H_1(0) = 1, \quad H_1(\infty) = 0. \end{aligned} \tag{3.22}$$

Second order $O(\varepsilon^2)$:

$$F_2''' + F_2'' + F_1 F_1'' - \frac{M}{1+m^2}(F_1' + mE_1') + G_r G_1 + G_m H_1 = 0 \tag{3.23}$$

$$E_2''' + E_2'' + F_1 E_1'' + \frac{M}{1+m^2}(mF_1' - E_1') = 0 \tag{3.24}$$

$$\begin{aligned} &G_2'' + P_r(G_2' + F_1 G_1') - SP_r G_1 + J_h P_r (2F_1'' F_2'' + 2E_1'' E_2'') \\ &\quad + \frac{M J_h P_r}{1+m^2}(F_1'^2 + E_1'^2) = 0 \end{aligned} \tag{3.25}$$

$$H_2'' + S_c(H_2' + F_1 H_1') + S_c S_0 G_2'' = 0 \tag{3.26}$$

$$\begin{aligned} &F_2(0) = 0, \quad F_2'(0) = 0, \quad F_2'(\infty) = 0, \quad E_2(0) = 0, \quad E_2'(0) = 0, \\ &E_2'(\infty) = 0, \quad G_2(0) = 0, \quad G_2(\infty) = 0, \quad H_2(0) = 0, \quad H_2(\infty) = 0. \end{aligned} \tag{3.27}$$

The solutions of the above coupled equations under corresponding boundary conditions are:

$$F_1 = 1 - e^{-\xi} \tag{3.28}$$

$$E_1 = 1 - e^{-\xi} \tag{3.29}$$

$$G_1 = (1 + A_1)e^{-P_r \xi} - A_1 e^{-2\xi} \tag{3.30}$$

$$H_1 = (1 - A_4 - A_5)e^{-S_c \xi} + A_4 e^{-P_r \xi} + A_5 e^{-2\xi} \tag{3.31}$$

$$F_2 = A_{13} + A_{12}e^{-\xi} + A_7\xi e^{-\xi} - \frac{A_8}{4}e^{-2\xi} - A_{10}e^{-P_r\xi} - A_{11}e^{-S_c\xi} \tag{3.32}$$

$$E_2 = \left(\frac{1}{4} - A_{14}\right) + \left(A_{14} - \frac{1}{2}\right)e^{-\xi} + A_{14}\xi e^{-\xi} + \frac{1}{4}e^{-2\xi} \tag{3.33}$$

$$\begin{aligned} G_2 = {} & A_{24}e^{-P_r\xi} - A_{15}\xi e^{-P_r\xi} + A_{21}\xi e^{-2\xi} - A_{22}e^{-2\xi} - A_{23}e^{-(1+S_c)\xi} \\ & + \frac{A_{16}}{2(2-P_r)}e^{-2\xi} - \frac{A_{17}}{1+P_r}e^{-(1+P_r)\xi} + \frac{A_{18}}{3(3-P_r)}e^{-3\xi} \end{aligned} \tag{3.34}$$

$$\begin{aligned} H_2 = {} & A_{37}e^{-S_c\xi} + A_{32}e^{-(1+P_r)\xi} + A_{33}\xi e^{-P_r\xi} - A_{34}e^{-P_r\xi} \\ & - A_{35}\xi e^{-2\xi} + A_{36}e^{-2\xi} - A_6 S_c\xi e^{-S_c\xi} + \frac{A_{25}}{2-S_c}e^{-2\xi} \\ & + \frac{A_{26}}{(P_r - S_c)}e^{-P_r\xi} - \frac{A_{27}}{3(3-S_c)}e^{-3\xi} + \frac{A_{28}}{1+S_c}e^{-(1+S_c)\xi} \end{aligned} \tag{3.35}$$

Hence we obtain the velocity, the temperature and the concentration fields as follows:

$$u = U_0 f'(\eta) = U_0[F_1'(\xi) + \varepsilon F_2'(\xi)] \tag{3.36}$$

$$w = U_0 g'(\eta) = U_0[E_1'(\xi) + \varepsilon E_2'(\xi)] \tag{3.37}$$

$$\theta(\eta) = G_1(\xi) + \varepsilon G_2(\xi) \tag{3.38}$$

$$\phi(\eta) = H_1(\xi) + \varepsilon H_2(\xi). \tag{3.39}$$

4. Skin-friction, nusselt number and sherwood number. The quantities of chief physical interests are the skin-friction due to primary velocity, the skin-friction due to secondary velocity, the Nusselt number and the Sherwood number.

The equation defining the wall skin-friction due to primary velocity (τ_p) is:

$$\tau_p = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}. \tag{4.1}$$

Thus from equation (4.1), we have:

$$\begin{aligned}\tau_p \propto [f''(\eta)]_{\eta=0} &= -1+\varepsilon\left[B_3-\frac{B_4}{1-P_r}+B_5+\frac{G_m(B_1+B_2-1)}{1-S_c}-2A_3\right]\\ &+\varepsilon^2\Big[S_c^2A_{54}+P_r^2A_{53}-4A_{52}-2A_{51}(1-S_c)-2P_rA_{49}\\ &-(1+S_c)^2A_{48}-(1+P_r)^2A_{47}-\frac{1}{2}A_{42}+A_{39}-A_{38}\Big].\end{aligned} \tag{4.2}$$

The equation defining the wall skin-friction due to secondary velocity (τ_s) is:

$$\tau_s = \mu\left(\frac{\partial w}{\partial y}\right)_{y=0}. \tag{4.3}$$

Thus from equation (4.3), we have:

$$\begin{aligned}\tau_s \propto [g''(\eta)]_{\eta=0} &= -1+\varepsilon\left[\frac{1}{2}-A_2\right]+\varepsilon^2\Big[A_{34}+S_c^2A_{33}\\ &-P_r^2A_{32}+(1+S_c)^2A_{31}-(1+P_r)^2A_{30}\\ &+5A_{25}-\frac{1}{2}A_{24}-A_{23}-A_{22}+2A_{21}\Big].\end{aligned} \tag{4.4}$$

The rate of heat transfer in terms of Nusselt number (N_u) is:

$$N_u = -\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{4.5}$$

$$\begin{aligned}N_u \propto [\theta'']_{\eta=0} &= -(1+A_1)P_r+2A_1+\varepsilon\Big[-A_{15}-\frac{P_rA_{16}}{2-P_r}+A_{17}\\ &-\frac{A_{18}}{3-P_r}+A_{21}+2A_{22}+(1+S_c)A_{23}-P_rA_{24}\Big].\end{aligned} \tag{4.6}$$

The rate of mass transfer in terms of Sherwood number (S_h) is:

$$S_h = -\left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{4.7}$$

$$S_h \propto [\phi'']_{\eta=0} = -A_4P_r - 2A_5 - S_cA_6 + \varepsilon\Big[-S_cA_{37} - \frac{2A_{25}}{2-S_c} - \frac{P_rA_{26}}{P_r-S_c} - A_6S_c + A_{33} + P_rA_{34} - A_{35} - 2A_{36} + \frac{A_{27}}{3-S_c} - A_{28} - (1+P_r)A_{32}\Big]. \tag{4.8}$$

Table 1. Skin-friction due to primary velocity (τ_p).

P_r	S_c	M	G_r	G_m	m	S_0	J_h	S	f_w	τ_p
0.71	0.60	3.0	5.0	4.0	0.5	1.0	0.2	1.0	3.0	0.75076
7.00	0.60	3.0	5.0	4.0	0.5	1.0	0.2	1.0	3.0	-0.79101
0.71	0.78	3.0	5.0	4.0	0.5	1.0	0.2	1.0	3.0	65.1430
0.71	0.60	5.0	5.0	4.0	0.5	1.0	0.2	1.0	3.0	0.48411
0.71	0.60	3.0	8.0	4.0	0.5	1.0	0.2	1.0	3.0	1.77191
0.71	0.60	3.0	5.0	6.0	0.5	1.0	0.2	1.0	3.0	1.93595
0.71	0.60	3.0	5.0	4.0	0.8	1.0	0.2	1.0	3.0	0.78492
0.71	0.60	3.0	5.0	4.0	0.5	2.0	0.2	1.0	3.0	-0.94656
0.71	0.60	3.0	5.0	4.0	0.5	1.0	0.6	1.0	3.0	-0.32434
0.71	0.60	3.0	5.0	4.0	0.5	1.0	0.2	2.0	3.0	-0.79101
0.71	0.60	3.0	5.0	4.0	0.5	1.0	0.2	1.0	5.0	-0.92476

Table 2. Skin-friction due to secondary velocity (τ_s).

M	m	f_w	τ_s
3.0	0.5	3.0	-1.18878
5.0	0.5	3.0	-1.27765
1.0	0.5	3.0	-1.12314
3.0	0.8	3.0	-1.09569
3.0	0.2	3.0	-1.31126
3.0	0.5	5.0	-1.06856
3.0	0.5	7.0	-1.03483

Table 3. Rate of heat transfer in terms of Nusselt number (N_u).

P_r	S_c	M	G_r	G_m	m	S_0	J_h	S	f_w	N_u
0.71	0.60	3.0	5.0	4.0	0.5	1.0	0.2	1.0	3.0	-0.51818
7.00	0.60	3.0	5.0	4.0	0.5	1.0	0.2	1.0	3.0	32.4863
0.71	0.78	3.0	5.0	4.0	0.5	1.0	0.2	1.0	3.0	3.05833
0.71	0.60	5.0	5.0	4.0	0.5	1.0	0.2	1.0	3.0	-0.44323
0.71	0.60	3.0	8.0	4.0	0.5	1.0	0.2	1.0	3.0	-0.60767
0.71	0.60	3.0	5.0	6.0	0.5	1.0	0.2	1.0	3.0	-1.66434
0.71	0.60	3.0	5.0	4.0	0.8	1.0	0.2	1.0	3.0	-0.54492
0.71	0.60	3.0	5.0	4.0	0.5	2.0	0.2	1.0	3.0	32.3491
0.71	0.60	3.0	5.0	4.0	0.5	1.0	0.6	1.0	3.0	-5.37361
0.71	0.60	3.0	5.0	4.0	0.5	1.0	0.2	2.0	3.0	33.2151
0.71	0.60	3.0	5.0	4.0	0.5	1.0	0.2	1.0	5.0	33.9158

Table 4. Rate of mass transfer in terms of Sherwood number (S_h).

P_r	S_c	M	G_r	G_m	m	S_0	J_h	S	f_w	S_h
0.71	0.60	3.0	5.0	4.0	0.5	1.0	0.2	1.0	3.0	0.46821
7.00	0.60	3.0	5.0	4.0	0.5	1.0	0.2	1.0	3.0	4.03817
0.71	0.78	3.0	5.0	4.0	0.5	1.0	0.2	1.0	3.0	-2.17766
0.71	0.60	5.0	5.0	4.0	0.5	1.0	0.2	1.0	3.0	0.42323
0.71	0.60	3.0	8.0	4.0	0.5	1.0	0.2	1.0	3.0	0.52189
0.71	0.60	3.0	5.0	6.0	0.5	1.0	0.2	1.0	3.0	1.15589
0.71	0.60	3.0	5.0	4.0	0.8	1.0	0.2	1.0	3.0	0.48424
0.71	0.60	3.0	5.0	4.0	0.5	2.0	0.2	1.0	3.0	8.88252
0.71	0.60	3.0	5.0	4.0	0.5	1.0	0.6	1.0	3.0	7.94100
0.71	0.60	3.0	5.0	4.0	0.5	1.0	0.2	2.0	3.0	3.60083
0.71	0.60	3.0	5.0	4.0	0.5	1.0	0.2	1.0	5.0	3.22014

5. Discussion. In order to get physical insight into the problem the effects of Prandtl number (P_r), Schmidt number (S_c), magnetic parameter (M), Grashof number (G_r), modified Grashof number (G_m), Hall parameter

(m), Soret number (S_0), heat source parameter (S), current density (J_h) and suction parameter (f_w) on primary velocity field (u), secondary velocity field (w), temperature field (T), concentration field (C), primary skin-friction (τ_p), secondary skin-friction (τ_s), the rate of heat transfer in terms of Nusselt number (N_u) and the rate of mass transfer in terms of Sherwood number (S_h) are studied taking numerical values. To observe the effects of these parameters, the values of Schmidt number (S_c) are chosen for water-vapour ($S_c = 0.60$) and Ammonia ($S_c = 0.78$) which represent diffusing chemical species of most common interest in air. The values of Prandtl number (P_r) are chosen to be $P_r = 0.71$ and $P_r = 7.00$ which correspond to air and water at 20^0C respectively.

Figure 1 shows the effects of Prandtl number (P_r), magnetic parameter (M), modified Grashof number (G_m), Hall parameter (m) and suction parameter (f_w) on primary velocity profiles (u) at $G_r = 5.0$, $S_0 = 1.0$, $J_h = 0.2$, $S_c = 0.60$ and $S = 1.0$ while Figure 2 shows the effects of Schmidt number (S_c), Grashof number (G_r), Soret number (S_0) and current density (J_h) on primary velocity profiles (u) at $P_r = 0.71$, $M = 3.0$, $G_m = 4.0$, $m = 0.5$, $S = 1.0$ and $f_w = 3.0$. Figure 3 shows the effects of magnetic parameter (M), Hall parameter (m) and suction parameter (f_w) on secondary velocity profiles (w). Figure 4 shows the effects of Prandtl number (P_r), Grashof number (G_r), modified Grashof number (G_m), Soret number (S_0) and current density (J_h) on temperature field (T) at $M = 3.0$, $m = 0.5$, $S_c = 0.60$, $S = 1.0$ and $f_w = 3.0$ while Figure 5 shows the effects of Schmidt number (S_c), Magnetic parameter (M), Hall parameter (m), heat source parameter (S) and suction parameter (f_w) on temperature field (T) at $P_r = 0.71$, $G_r = 5.0$, $G_m = 4.0$, $S_0 = 1.0$ and $J_h = 0.2$. Figure 6 shows the effect of Prandtl number (P_r), Schmidt number (S_c), magnetic parameter (M), Grashof number (G_r) and current density (J_h) on concentration fields (C) at $m = 0.5$, $G_m = 4.0$, $S_0 = 1.0$, $S = 1.0$ and $f_w = 3.0$ while Figure 7 shows the effect of Hall parameter (m), Soret number (S_0), heat source parameter (S), modified grashof number (G_m) and suction parameter (f_w) on concentrations fields (C) at $M = 3.0$, $P_r = 0.71$, $G_r = 5.0$, $S_c = 0.60$ and $J_h = 0.2$.

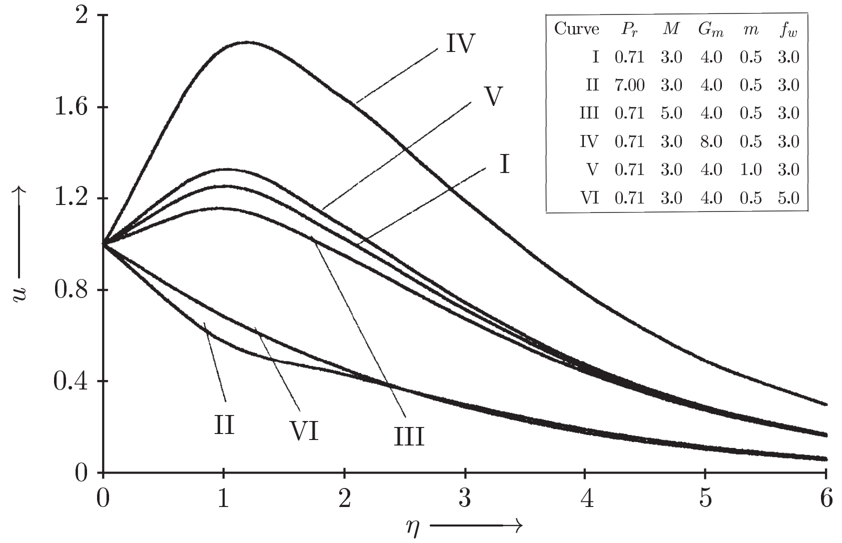


Figure 1. Effects of P_r, M, G_m, m and f_w on primary velocity field.

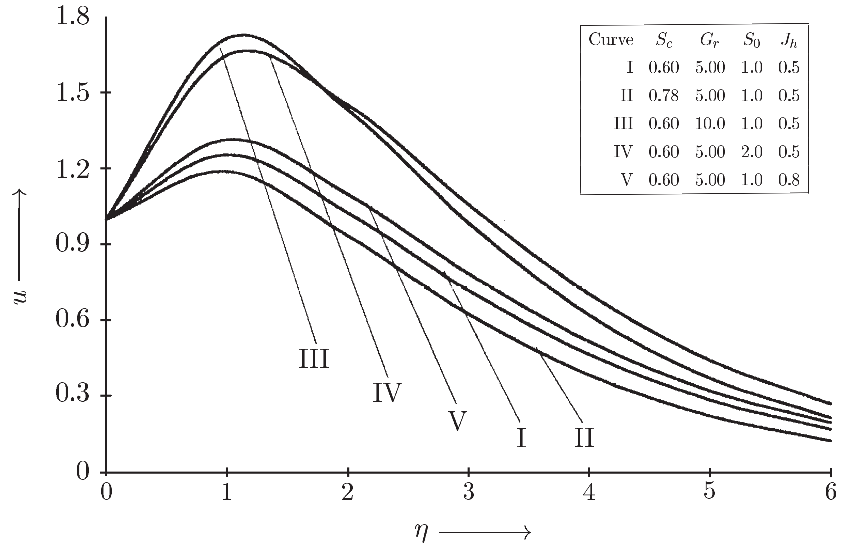


Figure 2. Effects of S_c, G_r, S_0 and J_h on primary velocity field.

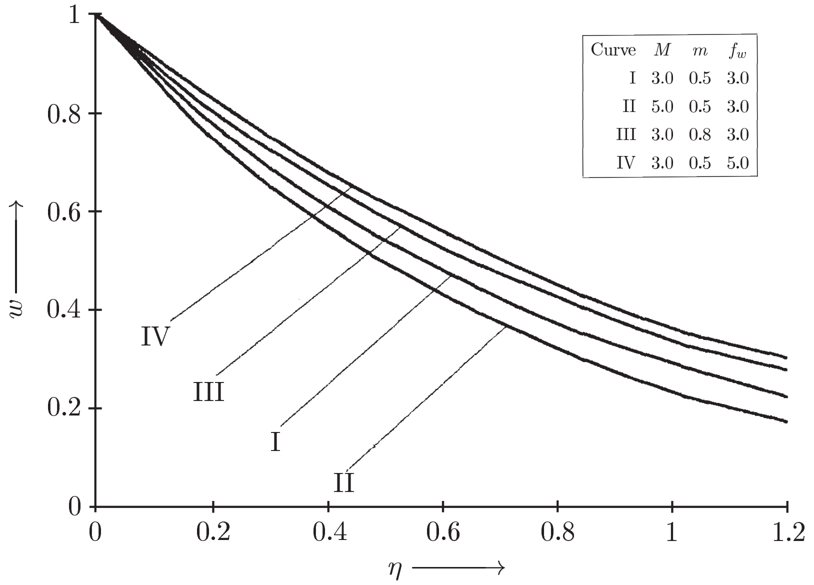


Figure 3. Effects of M, m and f_w on secondary velocity field.

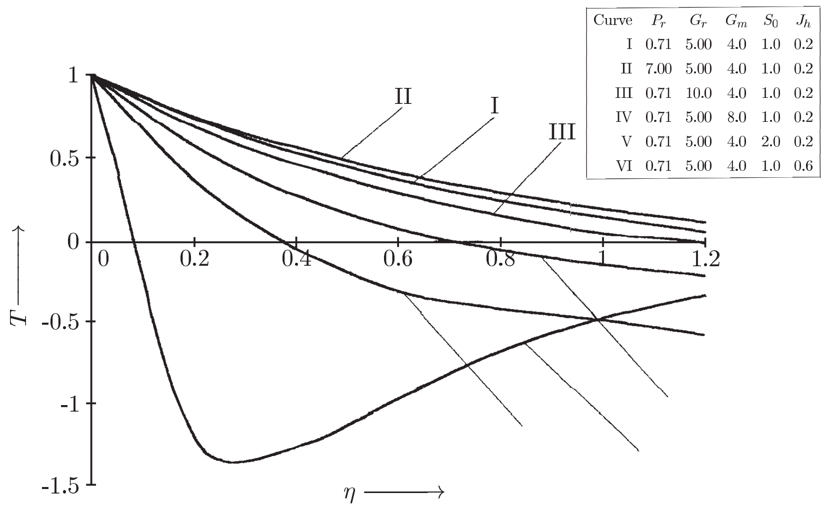
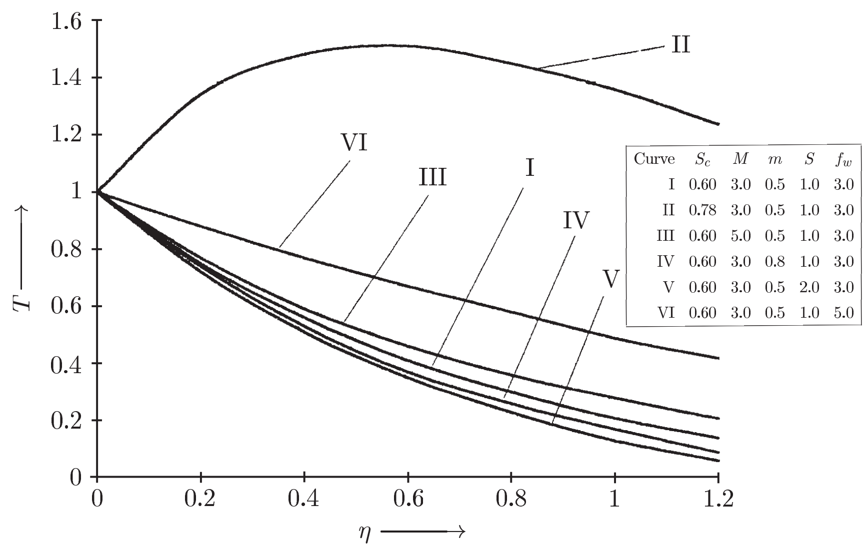
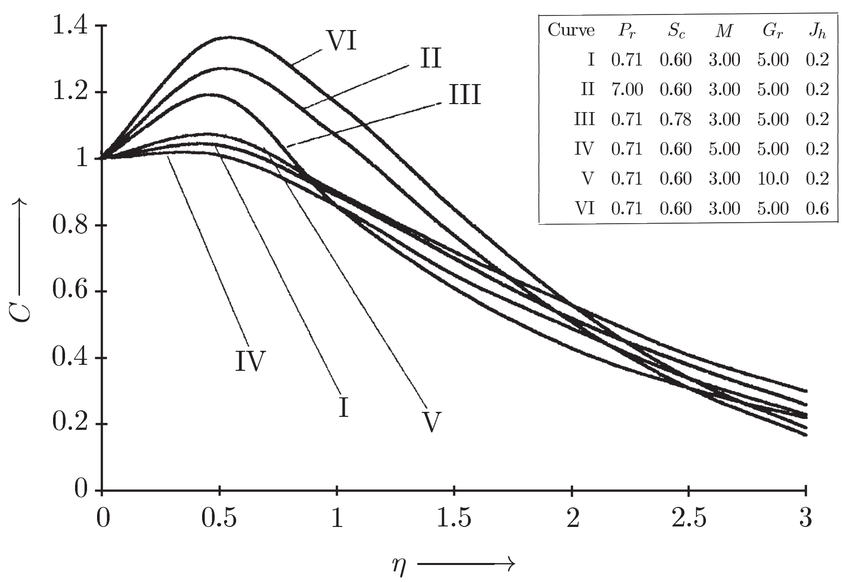


Figure 4. Effects of P_r, G_r, G_m, S_0 and J_h on temperature field.



Curve	S_c	M	m	S	f_w
I	0.60	3.0	0.5	1.0	3.0
II	0.78	3.0	0.5	1.0	3.0
III	0.60	5.0	0.5	1.0	3.0
IV	0.60	3.0	0.8	1.0	3.0
V	0.60	3.0	0.5	2.0	3.0
VI	0.60	3.0	0.5	1.0	5.0

Figure 5. Effects of S_c, M, m, S and f_w on temperature field.



Curve	P_r	S_c	M	G_r	J_h
I	0.71	0.60	3.00	5.00	0.2
II	7.00	0.60	3.00	5.00	0.2
III	0.71	0.78	3.00	5.00	0.2
IV	0.71	0.60	5.00	5.00	0.2
V	0.71	0.60	3.00	10.0	0.2
VI	0.71	0.60	3.00	5.00	0.6

Figure 6. Effects of P_r, S_c, M, G_r and J_h on concentration field.

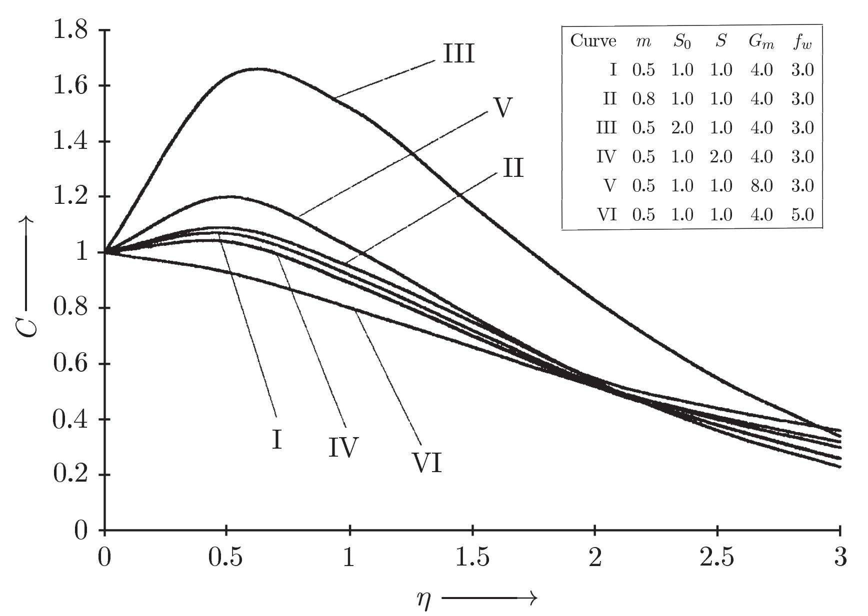


Figure 7. Effects of m, S_0, S, G_m and f_w on concentration field.

Table 1 represents the effects of P_r, S_c, M, G_r, G_m, m, S_0, S, J_h and f_w on the skin-friction (τ_p) due to primary velocity. Table 2 represents the effects of M, m and f_w on the skin-friction (τ_s) due to secondary velocity. Table 3 represents the effects of P_r, S_c, M, G_r, G_m, m, S_0, S, J_h and f_w on the rate of mass transfer in terms of Nusselt number (N_u). Table 4 represents the effects of P_r, S_c, M, G_r, G_m, m, S_0, S, J_h and f_w on the rate of mass transfer in terms of Sherwood number (S_h).

6. Conclusions. The conclusions of the study are as follows:

(i) An increase in P_r, f_w or M results in a decrease in the primary velocity field (u) while an increase in m or G_m results in an increase the primary velocity field (u).

(ii) An increase in G_r, S_0 or J_h leads to an increase in the primary velocity field (u) while an increase S_c in leads to a decrease in primary velocity

(u).

(iii) The presence as well as an increase in M decreases the secondary velocity field (w) while an increase in m or f_w increases the secondary velocity field (w).

(iv) An increase in P_r results in an increase in the temperature field (T) while an increase in G_r, G_m, S_0 or J_h results in a decrease in the temperature field (T).

(v) An increase in m or S decreases the temperature field (T) while an increase in S_c, M or f_w increases the temperature field (T).

(vi) An increase in M results in a decrease in the concentration field (C) while an increase in P_r, S_c, G_r or J_h results in an increase in the concentration field (C).

(vii) An increase in m, S_0 or G_m increases the concentration field (C) while an increase in S or f_w decreases concentration field (C).

(viii) An increase in S_c, G_r, G_m or m leads to an increase in the primary skin-friction (τ_p) while an increase in P_r, M, S_0, S, J_h or f_w decreases the primary skin-friction (τ_p).

(ix) An increase in M leads to a decrease in the secondary skin-friction (τ_s) while an increase in m or f_w increases the secondary skin-friction (τ_s).

(x) An increase in P_r, S_c, M, S_0, J_h or f_w leads to an increase in the Nusselt number (N_u) while reverse effect is noted for an increase in G_r, G_m, m or S.

(xi) An increase in P_r, M, G_r, G_m, S_0, S, J_h or f_w leads to an increase in the Sherwood number (S_h) while reverse effect is noted for an increase in S_c or m.

Appendix.

$$A_1 = \frac{J_h P_r}{2 - P_r}, \quad A_2 = S_0 S_c P_r (1 + A_1), \quad A_3 = 2 S_0 S_c A_1,$$

$$A_4 = \frac{A_2}{S_c - P_r}, \quad A_5 = \frac{A_3}{2 - S_c}, \quad A_6 = 1 - A_4 - A_5,$$

$$A_7 = 1 + \frac{M(1+m)}{1+m^2}, \quad A_8 = G_r A_1 - G_m A_5 - 1, \quad A_9 = G_r (1 + A_1) + G_m A_4,$$

$$A_{10} = \frac{A_9}{P_r^2(1-P_r)}, \qquad A_{11} = \frac{G_m A_6}{S_c^2(1-S_c)},$$

$$A_{12} = A_7 + \frac{A_8}{2} + P_r A_{10} + S_c A_{11}, \qquad A_{13} = \frac{A_8}{2} + A_{10} + A_{11} - A_{12},$$

$$A_{14} = 1 + \frac{M(1-m)}{1+m^2}, \qquad A_{15} = (P_r + S)(1 + A_1),$$

$$A_{16} = P_r \left[2J_h(A_{12} - 2A_7 - A_{14} - 0.5) - A_1 S - 2A_1 - \frac{2MJ_h}{1+m^2}\right],$$

$$A_{17} = P_r^2(1 + A_1) + 2J_h P_r^3 A_{10}, \qquad A_{18} = 2A_1 P_r + 2J_h P_r(1 - A_8),$$

$$A_{19} = 2J_h P_r(A_7 + A_{14}), \qquad A_{20} = 2J_h P_r S_c^2 A_{11},$$

$$A_{21} = \frac{A_{19}}{4 - 2P_r}, \qquad A_{22} = \frac{A_{21}(P_r - 4)}{4 - 2P_r},$$

$$A_{23} = \frac{A_{20}}{(1 + S_c - P_r)(1 + S_c)},$$

$$A_{24} = A_{22} + A_{23} - \frac{A_{16}}{2(2-P_r)} + \frac{A_{17}}{1+P_r} - \frac{A_{18}}{3(3-P_r)},$$

$$A_{25} = S_c \left[A_5 + S_0 \left(2A_{22} + 2A_{21} - \frac{A_{16}}{2(2-P_r)}\right)\right],$$

$$A_{26} = S_c A_4 - S_c S_0(2A_{15} + P_r A_{24}), \qquad A_{27} = S_c \left[2A_5 + \frac{3S_0 A_{18}}{3 - P_r}\right],$$

$$A_{28} = S_c S_0 (1 + S_c)^2 A_{23} - S_c^2 A_6, \qquad A_{29} = S_c S_0(1 + P_r)A_{17} - P_r S_c A_4,$$

$$A_{30} = S_c S_0 P_r^2 A_{15}, \qquad A_{31} = 4S_c S_0 A_{21},$$

$$A_{32} = \frac{A_{29}}{(1+P_r)(1+P_r-S_c)}, \qquad A_{33} = \frac{A_{30}}{P_r(P_r - S_c)},$$

$$A_{34} = \frac{A_{33}(S_c - 2P_r)}{P_r(P_r - S_c)}, \quad A_{35} = \frac{A_{31}}{2(2-S_c)}, \quad A_{36} = \frac{A_{35}(S_c - 4)}{2(2-S_c)},$$

and
$$A_{37} = \frac{A_{25}}{(2-S_c)} - \frac{A_{26}}{P_r - S_c} + \frac{A_{27}}{3(3-S_c)} - \frac{A_{28}}{1+S_c} - A_{32} + A_{34} - A_{36}$$

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Department of Mathematics, V. S. S. D. College, Kanpur - 208 012, U. P., India.

Department of Mathematics, C. L. Jain (P. G.) College, FIROZABAD - 283203, U. P., India.

236, Durga Nagar, Firozabad - 283203, U. P., India.