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GENERALIZATION OF SOME FUZZY FUNCTIONS

BY

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Abstract. The purpose of this paper, by introducing the notion of fuzzy slightly precontinuity, generalize fuzzy precontinuity and some types of fuzzy continuity which have been unified in [2] including fuzzy continuity, fuzzy weakly continuity, fuzzy θ-continuity, fuzzy strongly θ-continuity, fuzzy almost strongly θ-continuity, fuzzy weakly θ-continuity, fuzzy almost continuity, fuzzy super continuity and fuzzy δ-continuity. Furthermore, basic properties and preservation therems of fuzzy slightly precontinuous are obtained.

1. Introduction. Shahna [14] introduced the concept of fuzzy precontinuity in 1991 and Ekici [2] has introduced a unification for some types of continuous fuzzy functions in 2004.

In this paper, the notion of fuzzy slightly precontinuity which generalize fuzzy precontinuity and the unification in [2] including fuzzy continuity [10], fuzzy weakly continuity [1], fuzzy θ-continuity [5, 8], fuzzy strongly θ-continuity [6,7], fuzzy almost strongly θ-continuity [8], fuzzy weakly θ-continuity [8], fuzzy almost continuity [1], fuzzy super continuity [7] and fuzzy δ-continuity [3, 12] is introduced. Moreover, basic properties and preservation therems of fuzzy slightly precontinuous are obtained.

Firstly, characterizations and basic properties of fuzzy slightly precontin-

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uous functions are obtained. Secondly, relationships between fuzzy slightly precontinuity and separation axioms are investigated. Furthermore, the notion of fuzzy pre-co-closed graphs is introduced and the relationships between fuzzy slightly precontinuity and fuzzy pre-co-closed graphs are investigated. In the last section, relationships between fuzzy slightly precontinuity and compactness and between fuzzy slightly precontinuity and connectedness are investigated.

2. Preliminaries. Throughout this paper, X and Y are fuzzy topological spaces.

The class of fuzzy sets on a universe X will be denoted by I^X and fuzzy sets on X will be denoted by Greek letters as μ, ρ, η, etc. A family τ of fuzzy sets in X is called a fuzzy topology for X iff (1) $\emptyset$, $X \in \tau$, (2) $\mu \wedge \rho \in \tau$ whenever μ, $\rho \in \tau$ and (3) If $\mu_i \in \tau$ for each $i \in I$, then $\vee \mu_i \in \tau$. Moreover, the pair (X, τ) is called a fuzzy topological space. Every member of τ is called a open fuzzy set [9].

Let μ be a fuzzy set in X. We denote the interior and the closure of a fuzzy set μ by $int(\mu)$ and $cl(\mu)$, respectively.

A fuzzy set in X is called a fuzzy point iff it takes the value 0 for all $y \in X$ except one, say, $x \in X$. If its value at x is α $(0 < \alpha \leq 1)$ we denote this fuzzy point by x_α, where the point x is called its support [9]. For any fuzzy point x_ε and any fuzzy set μ, we write $x_\varepsilon \in \mu$ iff $\varepsilon \leq \mu(x)$.

A fuzzy set μ in a space X is called fuzzy preopen [14] if $\mu \leq int(cl(\mu))$. The complement of a fuzzy preopen set is said to be preclosed.

Let $f : X \to Y$ a fuzzy function from a fuzzy topological space X to a fuzzy topological space Y. Then the function $g : X \to X \times Y$ defined by $g(x_\varepsilon) = (x_\varepsilon, f(x_\varepsilon))$ is called the fuzzy graph function of f [1].

Definition 1. A fuzzy function $f : X \to Y$ is said to be fuzzy precontinuous [14] if $f^{-1}(\beta)$ is fuzzy preopen set in X for every fuzzy open set β in Y.

3. Fuzzy slightly precontinuous functions. It is known that a fuzzy set is called fuzzy clopen iff it is both fuzzy open and fuzzy closed.

Definition 2. Let (X, τ) and (Y, υ) be fuzzy topological spaces. A fuzzy function $f : X \to Y$ is said to be fuzzy slightly precontinuous if for each fuzzy point $x_\varepsilon \in X$ and each fuzzy clopen set β in Y containing $f(x_\varepsilon)$, there exists a fuzzy preopen set μ in X containing x_ε such that $f(\mu) \leq \beta$.

Theorem 3. *For a function $f : X \to Y$, the following statements are equivalent:*

(1) *f is fuzzy slightly precontinuous;*
(2) *for every fuzzy clopen set β in Y, $f^{-1}(\beta)$ is fuzzy preopen;*
(3) *for every fuzzy clopen set β in Y, $f^{-1}(\beta)$ is fuzzy preclosed;*
(4) *for every fuzzy clopen set β in Y, $f^{-1}(\beta)$ is fuzzy preclopen.*

Proof. (1) $\Rightarrow$ (2) : Let β be a fuzzy clopen set in Y and let $x_\varepsilon \in f^{-1}(\beta)$. Since $f(x_\varepsilon) \in \beta$, by (1), there exists a fuzzy preopen set μ_{x_ε} in X containing x_ε such that $\mu_{x_\varepsilon} \leq f^{-1}(\beta)$. We obtain that $f^{-1}(\beta) = \bigvee_{x_\varepsilon \in f^{-1}(\beta)} \mu_{x_\varepsilon}$. Thus, $f^{-1}(\beta)$ is fuzzy preopen.

(2) $\Rightarrow$ (3) : Let β be a fuzzy clopen set in Y. Then, $Y \backslash \beta$ is fuzzy clopen. By (2), $f^{-1}(Y \backslash \beta) = X \backslash f^{-1}(\beta)$ is fuzzy preopen. Thus, $f^{-1}(\beta)$ is fuzzy preclosed.

(3) $\Rightarrow$ (4) : It can be shown easily.

(4) $\Rightarrow$ (1) : Let β be a fuzzy clopen set in Y containing $f(x_\varepsilon)$. By (4), $f^{-1}(\beta)$ is preclopen. If we take $\mu = f^{-1}(\beta)$, then $f(\mu) \leq \beta$. Hence, f is fuzzy slightly precontinuous.

Theorem 4. *Let $f : X \to Y$ be a fuzzy function and let $g : X \to X \times Y$ be the fuzzy graph function of f, defined by $g(x_\varepsilon) = (x_\varepsilon, f(x_\varepsilon))$ for every $x_\varepsilon \in X$. If g is fuzzy slightly precontinuous, then f is fuzzy slightly precontinuous.*

Proof. Let β be a fuzzy clopen set in Y, then $X \times \beta$ is a fuzzy clopen set in $X \times Y$. Since g is fuzzy slightly precontinuous, then $f^{-1}(\beta) = g^{-1}(X \times \beta)$ is fuzzy preopen in X. Thus, f is fuzzy slightly precontinuous.

Definition 5. A fuzzy filter base Λ is said to be fuzzy p-convergent to a fuzzy point x_ε in X if for any fuzzy preopen set β in X containing x_ε, there exists a fuzzy set $\mu \in \Lambda$ such that $\mu \leq \beta$.

Definition 6. A fuzzy filter base Λ is said to be fuzzy co-convergent to a fuzzy point x_ε in X if for any fuzzy clopen set β in X containing x_ε, there exists a fuzzy set $\mu \in \Lambda$ such that $\mu \leq \beta$.

Theorem 7. *If a fuzzy function $f : X \to Y$ is fuzzy slightly precontinuous, then for each fuzzy point $x_\varepsilon \in X$ and each fuzzy filter base Λ in X p-converging to x_ε, the fuzzy filter base $f(\Lambda)$ is fuzzy co-convergent to $f(x_\varepsilon)$.*

Proof. Let $x_\varepsilon \in X$ and Λ be any fuzzy filter base in X p-converging to x_ε. Since f is fuzzy slightly precontinuous, then for any fuzzy clopen set λ in Y containing $f(x_\varepsilon)$, there exists a fuzzy preopen set μ in X containing x_ε such that $f(\mu) \leq \lambda$. Since Λ is fuzzy p-converging to x_ε, there exists a $B \in \Lambda$ such that $\beta \leq \mu$. This means that $f(\beta) \leq \lambda$ and therefore the fuzzy filter base $f(\Lambda)$ is fuzzy co-convergent to $f(x_\varepsilon)$.

Remark 8. Obviously this paper generalize Ekici's paper [2]. We obtain that fuzzy $\varphi\psi$-continuity [2] implies fuzzy slightly precontinuity. If we take $\varphi = cl$, $\psi = i$, then the following example shows that this implication is not reversible.

Example 9. Let $X = \{x, y, z\}$ and μ, β, ρ be fuzzy sets of X defined as follows:

$$\mu(x) = 0,3 \quad \mu(y) = 0,2 \quad \mu(z) = 0,7$$
$$\beta(x) = 0,8 \quad \beta(y) = 0,8 \quad \beta(z) = 0,4$$
$$\rho(x) = 0,8 \quad \rho(y) = 0,7 \quad \rho(z) = 0,6$$

We put $\tau_1 = \{X, \emptyset, \mu, X \backslash \mu\}$, $\tau_2 = \{X, \emptyset, \rho\}$ and let $f : (X, \tau_1) \to (X, \tau_2)$ be a fuzzy identity function. Then, f is fuzzy slightly precontinuous, but it is not fuzzy strongly θ-continuous.

Remark 10. Obviously fuzzy precontinuity implies fuzzy slightly precontinuity. The following example shows that this implication is not reversible.

Example 11. Let $X = \{a, b\}$, $Y = \{x, y\}$ and λ, μ are fuzzy sets defined as follows:

$$\lambda(a) = 0,3 \quad \lambda(b) = 0,4$$
$$\mu(x) = 0,7 \quad \mu(y) = 0,5$$

Let $\tau_1 = \{X, \emptyset, \lambda\}$, $\tau_2 = \{Y, \emptyset, \mu\}$. Then the fuzzy function $f : (X, \tau_1) \to (Y, \tau_2)$ defined by $f(a) = x$, $f(b) = y$ is fuzzy slightly precontinuous but not fuzzy precontinuous.

Theorem 12. Suppose that Y has a base consisting of fuzzy clopen sets. If $f : X \to Y$ is fuzzy slightly precontinuous, then f is fuzzy precontinuous.

Proof. Let $x_\varepsilon \in X$ and let ρ be a fuzzy open set in Y containing $f(x_\varepsilon)$. Since Y has a base consisting of fuzzy clopen sets, there exists a fuzzy clopen set β containing $f(x_\varepsilon)$ such that $\beta \leq \rho$. Since f is fuzzy slightly precontinuous, then there exists a fuzzy preopen set μ in X containing x_ε such that $f(\mu) \leq \beta \leq \rho$. Thus, f is fuzzy precontinuous.

4. Fuzzy separation axioms and fuzzy pre-co-closed graphs. In this section, we investigate the relationships between fuzzy slightly precontinuous functions and separation axioms. Moreover, we introduce the notion of fuzzy pre-co-closed graphs and we investigate the relationships between fuzzy slightly precontinuity and fuzzy pre-co-closed graphs.

Definition 13. A fuzzy space X is said to be fuzzy p-T_1 if for each pair of distinct fuzzy points x_ε and y_ν of X, there exist fuzzy preopen sets β and μ containing x_ε and y_ν respectively such that $y_\nu \notin \beta$ and $x \notin \mu$.

Definition 14. A fuzzy space X is said to be fuzzy p-T_0 if for each pair of distinct fuzzy points in X, there exists a fuzzy preopen set of X containing one point but not the other.

Definition 15. A fuzzy space X is said to be fuzzy co-T_1 if for each pair of distinct fuzzy points x_ε and y_ν of X, there exist fuzzy clopen sets β and μ containing x_ε and y_ν respectively such that $y_\nu \notin \beta$ and $x_\varepsilon \notin \mu$.

Definition 16. A fuzzy space X is said to be fuzzy co-T_0 if for each pair of distinct fuzzy points in X, there exists a fuzzy clopen set of X containing one point but not the other.

Theorem 17. *If $f : X \to Y$ is a fuzzy slightly precontinuous injection and Y is fuzzy co-T_1, then X is fuzzy p-T_1.*

Proof. Suppose that Y is fuzzy co-T_1. For any distict fuzzy points x_ε and y_ν in X, there exist fuzzy clopen sets μ , ρ in Y such that $f(x_\varepsilon) \in \mu$, $f(y_\nu) \notin \mu$, $f(x_\varepsilon) \notin \rho$ and $f(y_\nu) \in \rho$. Since f is fuzzy slightly precontinuous, $f^{-1}(\mu)$ and $f^{-1}(\rho)$ are preopen sets in X such that $x_\varepsilon \in f^{-1}(\mu)$, $y_\nu \notin f^{-1}(\mu)$, $x_\varepsilon \notin f^{-1}(\rho)$ and $y_\nu \in f^{-1}(\rho)$. This shows that X is fuzzy p-T_1.

Definition 18. A fuzzy space X is said to be fuzzy p-T_2 (p-Hausdorff) if for each pair of distinct fuzzy points x_ε and y_ν in X, there exist disjoint fuzzy preopen sets β and μ in X such that $x_\varepsilon \in \beta$ and $y_\nu \in \mu$.

Definition 19. A fuzzy space X is said to be fuzzy co-T_2 (co-Hausdorff) if for each pair of distinct fuzzy points x_ε and y_ν in X, there exist disjoint fuzzy clopen sets β and μ in X such that $x_\varepsilon \in \beta$ and $y_\nu \in \mu$.

Theorem 20. *If $f : X \to Y$ is a fuzzy slightly precontinuous injection and Y is fuzzy co-T_2, then X is fuzzy p-T_2.*

Proof. For any pair of distict fuzzy points x_ε and y_ν in X, there exist disjoint fuzzy clopen sets β and μ in Y such that $f(x_\varepsilon) \in \beta$ and $f(y_\nu) \in \mu$. Since f is fuzzy slightly precontinuous, $f^{-1}(\beta)$ and $f^{-1}(\mu)$ is fuzzy preopen in X containing x_ε and y_ν respectively. We have $f^{-1}(\beta) \wedge f^{-1}(\mu) = \emptyset$. This shows that X is p-T_2.

Definition 21. A space is called fuzzy co-regular (respectively fuzzy strongly p-regular) if for each fuzzy clopen (respectively fuzzy p-closed) set η and each fuzzy point $x_\varepsilon \notin \eta$, there exist disjoint fuzzy open sets β and μ such that $\eta \leq \beta$ and $x_\varepsilon \in \mu$.

Definition 22. A fuzzy space is said to be fuzzy co-normal (respectively fuzzy strongly p-normal) if for every pair of disjoint fuzzy clopen (respectively fuzzy p-closed) sets η_1 and η_2 in X, there exist disjoint fuzzy open sets β and μ such that $\eta_1 \leq \beta$ and $\eta_2 \leq \mu$.

Theorem 23. *If f is fuzzy slightly p-continuous injective fuzzy open function from a fuzzy strongly p-regular space X onto a fuzzy space Y, then Y is fuzzy co-regular.*

Proof. Let η be fuzzy clopen set in Y and be $y_\varepsilon \notin \eta$. Take $y_\varepsilon = f(x_\varepsilon)$. Since f is fuzzy slightly precontinuous, $f^{-1}(\eta)$ is a fuzzy preclosed set. Take $\lambda = f^{-1}(\eta)$. We have $x_\varepsilon \notin \lambda$. Since X is fuzzy strongly p-regular, there exist disjoint fuzzy open sets β and μ such that $\lambda \leq \beta$ and $x_\varepsilon \in \mu$. We obtain that $\eta = f(\lambda) \leq f(\beta)$ and $y_\varepsilon = f(x_\varepsilon) \in f(\mu)$ such that $f(\beta)$ and $f(\mu)$ are disjoint fuzzy open sets. This shows that Y is fuzzy co-regular.

Theorem 24. *If f is fuzzy slightly precontinuous injective fuzzy open function from a fuzzy strongly p-normal space X onto a fuzzy space Y, then Y is fuzzy co-normal.*

Proof. Let η_1 and η_2 be disjoint fuzzy clopen sets in Y. Since f is fuzzy slightly precontinuous, $f^{-1}(\eta_1)$ and $f^{-1}(\eta_2)$ are fuzzy preclosed sets. Take $\beta = f^{-1}(\eta_1)$ and $\mu = f^{-1}(\eta_2)$. We have $\beta \wedge \mu = \emptyset$. Since X is fuzzy strongly p-normal, there exist disjoint fuzzy open sets λ and ρ such that $\beta \leq \lambda$ and $\mu \leq \rho$. We obtain that $\eta_1 = f(\beta) \leq f(\lambda)$ and $\eta_2 = f(\mu) \leq f(\rho)$ such that $f(\lambda)$ and $f(\rho)$ are disjoint fuzzy open sets. Thus, Y is fuzzy co-normal.

Recall that for a fuzzy function $f : X \to Y$, the subset $\{(x_\varepsilon, f(x_\varepsilon)) : x_\varepsilon \in X\} \leq X \times Y$ is called the graph of f and is denoted by $G(f)$.

Definition 25. A graph $G(f)$ of a fuzzy function $f : X \to Y$ is said to be fuzzy pre-co-closed if for each $(x_\varepsilon, y_\nu) \in (X \times Y)\backslash G(f)$, there exist a fuzzy preopen set β in X containing x_ε and a fuzzy clopen set μ in Y containing y_ν such that $(\beta \times \mu) \wedge G(f) = \emptyset$.

Lemma 26. *A graph $G(f)$ of a fuzzy function $f : X \to Y$ is fuzzy pre-co-closed in $X \times Y$ if and only if for each $(x_\varepsilon, y_\nu) \in (X \times Y)\backslash G(f)$, there exist a fuzzy preopen set β in X containing x_ε and a fuzzy clopen set μ in Y containing y_ν such that $f(\beta) \wedge \mu = \emptyset$.*

Theorem 27. *If $f : X \to Y$ is fuzzy slightly precontinuous and Y is fuzzy co-Hausdorff, then $G(f)$ is fuzzy pre-co-closed in $X \times Y$.*

Proof. Let $(x_\varepsilon, y_\nu) \in (X \times Y)\backslash G(f)$, then $f(x_\varepsilon) \neq y_\nu$. Since Y is fuzzy co-Hausdorff, there exist fuzzy clopen sets β and μ in Y with $f(x_\varepsilon) \in \beta$ and $y_\nu \in \mu$ such that $\beta \wedge \mu = \emptyset$. Since f is fuzzy slightly precontinuous, there exists a preopen set ρ in X containing x_ε such that $f(\rho) \leq \beta$. Therefore, we obtain $y_\nu \in \mu$ and $f(\rho) \wedge \mu = \emptyset$. This shows that $G(f)$ is fuzzy pre-co-closed.

Theorem 28. *If $f : X \to Y$ is fuzzy precontinuous and Y is fuzzy co-T_1, then $G(f)$ is fuzzy pre-co-closed in $X \times Y$.*

Proof. Let $(x_\varepsilon, y_\nu) \in (X \times Y)\backslash G(f)$, then $f(x_\varepsilon) \neq y_\nu$ and there exists a fuzzy clopen set μ in Y such that $f(x_\varepsilon) \in \mu$ and $y_\nu \notin \mu$. Since f is fuzzy precontinuous, there exists a preopen set β in X containing x_ε such that $f(\beta) \leq \mu$. Therefore, we obtain that $f(\beta) \wedge (Y\backslash\mu) = \emptyset$ and $Y\backslash\mu$ is fuzzy clopen containing y_ν. This shows that $G(f)$ is fuzzy pre-co-closed in $X \times Y$.

Theorem 29. *Let $f : X \to Y$ has a fuzzy pre-co-closed graph $G(f)$. If f is injective, then X is fuzzy p-T_1.*

Proof. Let x_ε and y_ν be any two distinct fuzzy points of X. Then, we have $(x_\varepsilon, f(y_\nu)) \in (X \times Y)\backslash G(f)$. By definition of fuzzy pre-co-closed graph, there exist a fuzzy preopen set β in X and a fuzzy clopen set μ in Y such that $x_\varepsilon \in \beta$, $f(y_\nu) \in \mu$ and $f(\beta) \wedge \mu = \emptyset$; hence $\beta \wedge f^{-1}(\mu) = \emptyset$. Therefore, we have $y_\nu \notin \beta$. This implies that X is fuzzy p-T_1.

Definition 30. A fuzzy function is called fuzzy M-preopen [13] if the image of each fuzzy preopen set in X is fuzzy preopen set in Y.

Theorem 31. *Let $f : X \to Y$ has a fuzzy pre-co-closed graph $G(f)$. If f is surective fuzzy M-preopen function, then Y is fuzzy p-T_2.*

Proof. Let y_ν and y_ξ be any distinct points of Y. Since f is surjective $f(x_\nu) = y_\nu$ for some $x_\nu \in X$ and $(x_\nu, y_\xi) \in (X \times Y)\backslash G(f)$. By fuzzy pre-co-closedness of graph $G(f)$, there exist a fuzzy preopen set β in X and a fuzzy clopen set μ in Y such that $x_\nu \in \beta$, $y_\xi \in \mu$ and $(\beta \times \mu) \wedge G(f) = \emptyset$. Then, we have $f(\beta) \wedge \mu = \emptyset$. Since f is fuzzy M-preopen, then $f(\beta)$ is fuzzy preopen such that $f(x_\nu) = y_\nu \in f(\beta)$. This implies that Y is fuzzy p-T_2.

5. Fuzzy covering properties and fuzzy connectedness. In this section, we investigate the relationships between fuzzy slightly precontinuous functions and fuzzy compactness and between fuzzy slightly precontinuous functions and fuzzy connectedness.

Definition 32. A fuzzy space X said to be

(1) fuzzy precompact [4] if every fuzzy preopen cover of X has a finite subcover.
(2) fuzzy countably precompact if every fuzzy preopen countably cover of X has a finite subcover.
(3) fuzzy pre-Lindelof if every cover of X by fuzzy preopen sets has a countable subcover.
(4) fuzzy mildly compact if every fuzzy clopen cover of X has a finite subcover.
(5) fuzzy mildly countably compact if every fuzzy clopen countably cover of X has a finite subcover.
(6) fuzzy mildly Lindelof if every cover of X by fuzzy clopen sets has a countable subcover.

Theorem 33. *Let $f : X \to Y$ be a fuzzy slightly precontinuous surjection. Then the following statements hold:*

(1) *if X is fuzzy precompact, then Y is fuzzy mildly compact.*
(2) *if X is fuzzy pre-Lindelof, then Y is fuzzy mildly Lindelof.*
(3) *if X is fuzzy countably precompact, then Y is fuzzy mildly countably compact.*

Proof. (1) Let $\{\mu_\alpha : \alpha \in I\}$ be any fuzzy clopen cover of Y. Since f is fuzzy slightly precontinuous, then $\{f^{-1}(\mu_\alpha) : \alpha \in I\}$ is a fuzzy preopen cover of X. Since X is fuzzy precompact, there exists a finite subset I_0 of I such that $X = \vee\{f^{-1}(\mu_\alpha) : \alpha \in I_0\}$. Thus, we have $Y = \vee\{\mu_\alpha : \alpha \in I_0\}$ and Y is fuzzy mildly compact.

The other proofs are smilarly.

Definition 34. A fuzzy space X said to be

(1) fuzzy preclosed-compact if every preclosed cover of X has a finite subcover.

(2) fuzzy countably preclosed-compact if every countable cover of X by preclosed sets has a finite subcover.

(3) fuzzy preclosed-Lindelof if every cover of X by preclosed sets has a countable subcover.

Theorem 35. *Let $f : X \to Y$ be a fuzzy slightly precontinuous surjection. Then the following statements hold:*

(1) *if X is fuzzy preclosed-compact, then Y is mildly compact.*

(2) *if X is fuzzy preclosed-Lindelof, then Y is mildly Lindelof.*

(3) *if X is fuzzy countably preclosed-compact, then Y is mildly countably compact.*

Proof. It can be obtained smilarly as the previous theorem.

Definition 36. A fuzzy space X is said to be fuzzy p-connected if it cannot be expressed as the union of two nonempty, disjoint fuzzy preopen sets.

Definition 37. A fuzzy space X is said to be fuzzy connected [11] if it cannot be expressed as the union of two nonempty, disjoint fuzzy open sets.

Theorem 38. *If $f : X \to Y$ is fuzzy slightly precontinuous surjective function and X is fuzzy p-connected space, then Y is fuzzy connected space.*

Proof. Suppose that Y is not fuzzy connected space. Then there exists nonempty disjoint fuzzy open sets β and μ such that $Y = \beta \vee \mu$. Therefore, β and μ are fuzzy clopen sets in Y. Since f is fuzzy slightly precontinuous, then

$f^{-1}(\beta)$ and $f^{-1}(\mu)$ are fuzzy preclosed and preopen in X. Moreover, $f^{-1}(\beta)$ and $f^{-1}(\mu)$ are nonempty disjoint and $X = f^{-1}(\beta) \vee f^{-1}(\mu)$. This shows that X is not fuzzy p-connected. This is a contradiction. By contradiction, Y is fuzzy connected.

Definition 39. A fuzzy space X is called hyperconnected if every fuzzy open set is dense.

Remark 40. The following example shows that fuzzy slightly precontinuous surjection do not necessarily preserve fuzzy hyperconnectedness.

Example 41. Let $X = \{x, y, z\}$ and μ, β, ρ be fuzzy sets of X defined as follows:

$$\begin{array}{lll} \mu(x) = 0,2 & \mu(y) = 0,2 & \mu(z) = 0,5 \\ \beta(x) = 0,8 & \beta(y) = 0,8 & \beta(z) = 0,4 \\ \rho(x) = 0,8 & \rho(y) = 0,7 & \rho(z) = 0,6 \end{array}$$

We put $\tau_1 = \{X, \emptyset, \rho\}$, $\tau_2 = \{X, \emptyset, \mu, \beta, \mu \wedge \beta, \mu \vee \beta\}$ and let $f : (X, \tau_1) \to (X, \tau_2)$ be a fuzzy identity function. Then f is fuzzy slightly precontinuous surjective.

(X, τ_1) is hyperconnected. But (X, τ_2) is not hyperconnected.

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