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ONE RELATOR QUOTIENTS OF THE HECKE GROUP $H(\frac{1+\sqrt{5}}{2})$

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Abstract. The group $H(\frac{1+\sqrt{5}}{2})$ is the next most well-worked Hecke group after the modular group generated by two rotations. In this paper we add an extra relation to the existing ones and obtain one relator quotients of $H(\frac{1+\sqrt{5}}{2})$. As Hecke groups act on sphere, all quotients are finite triangle groups.

1. Introduction. A Fuchsian group G is a finitely generated discrete subgroup of $PSL(2,R)$. There are four types of elements in a Fuchsian group G

$a_1, b_1, \ldots, a_g, b_g$	(hyperbolic)
$x_1, \ldots, x_r$	(elliptic)
$p_1, \ldots, p_t$	(parabolic)
$h_1, \ldots, h_u$	(hyperbolic boundary elements)

satisfying the relations

$$x_j^{m_j} = \prod_{i=1}^{g}[a_i, b_i] \prod_{j=1}^{r} x_j \prod_{k=1}^{t} p_k \prod_{l=1}^{u} h_l = 1$$

We then say G has signature $(g; m_1, \ldots, m_r; t; u)$; where $m_1, \ldots, m_r$ are integers ≥ 2 and are called the periods of G.

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Hecke groups $H(\lambda_q)$ are finitely generated discrete subgroups of $PSL(2, R)$, generated by

$$a(z) = -\frac{1}{z} \quad \text{and} \quad b(z) = -\frac{1}{z + \lambda_q}$$

where $\lambda_q = 2\cos\frac{\pi}{q}$, with $q \geq 3$ is an integer. For Hecke groups, $u = 0$, and we denote the signature briefly by $(0; 2, q; 1)$ as $H(\lambda_q)$ acts on sphere. The most important and well-worked Hecke group is the modular group obtained for $q = 3$ and therefore denoted by $H(\lambda_3) = H(1) = \Gamma$. Here the case $q = 5$, which shows many similarities to Γ, is considered. The Hecke group obtained in this case is $H(\lambda_5)$, where $\lambda_5 = \frac{1+\sqrt{5}}{2}$ is the golden ratio.

For the sake of brevity, we shall denote $a(z)$ and $b(z)$ with a and b.

In fact, the Hecke groups can be considered as special triangle groups with an infinity, in the following sense: Recall that a triangle group $T(l, m, n)$ is a two generator group with representation

$$< a, b : \ a^l = b^m = (ab)^n = 1 > .$$

It therefore has the signature $(0; 1, m, n)$. It is known that $T(l, m, n)$ is finite precisely when $\frac{1}{l} + \frac{1}{m} + \frac{1}{n} > 1$. In that case the triangle groups are as follows:

$T(1, n, n) \cong C_n$, the cyclic group of order n
$T(2, 2, n) \cong D_n$, the dihedral group of order $2n$
$T(2, 3, 3) \cong A_4$, the tetrahedral group of order 12
$T(2, 3, 4) \cong S_4$, the octahedral group of order 24
$T(2, 3, 5) \cong A_5$, the icosahedral group of order 60.

A generalized triangle group is defined as a group G_m with a presentation

$$G_m =< a, b : \ a^p = b^q = R^m(a, b) = 1 >$$

where $p \leq q$, $p \geq 2$ or $p = 0$, $q \geq 2$ or $q = 0$ and $R(a, b)$ is a cyclically reduced word in the free product on a and b involving both a and b and

$m \geq 2$. Here we define similar groups for $m = 1$ and consider

$$G_1 =< a, b : \ a^p = b^q = R(a, b) = 1 >$$

which can be thought of as one relator quotients of the free product $C_p * C_q$. In fact, G_m is a special case of G_1 since for example, a relator like $R(a, b) = ab^2ab^2ab^2$ can well be thought as $R^3(a, b) = (ab^2)^3$.

2. One Relator Quotients of $H(\frac{1+\sqrt{5}}{2})$. $H(\lambda_5)$ has a presentation $< a, b : a^2 = b^5 = 1 >$. We now add an extra relation

$$w = R(a, b) = 1$$

where $R(a, b)$ is a cyclically reduced word of the form

$$R(a, b) = ab^{\varepsilon_1}ab^{\varepsilon_2} \cdots ab^{\varepsilon_n}$$

with $1 \leq \varepsilon_i \leq 4$.

We denote the number of a's and b's in w by $e_a(w)$ and $e_b(w)$, respectively. Here note that $e_b(w) = \sum_{i=1}^{n} \varepsilon_i$.

Theorem 2.1. *If $e_a(w) = 0$, then $1 \leq e_b(w) \leq 4$ and if $e_a(w) = n$, then $n \leq e_b(w) \leq 4n$.*

Proof. Let $n > 0$. As for each i, ε_i could have four different values 1,2,3,4, the minimum value of $e_b(w)$ is n for the choice of $\varepsilon_i = 1$ for every i, and maximum value of $e_b(w)$ is $4n$, as the sum of n 4's.

If $n = 0$, then the word w equals to a power ε_i of b, implying the result.

We define $N_{k,l}$ as the total number of words w with $e_a(w) = k$ and $e_b(w) = l$. Then it follows that

Theorem 2.2. $N_{n,n} = N_{n,4n} = N_{n,n+1} = N_{n,4n-1} = 1$.

Proof follows by definition of w.

3. Tables. We now give the tables consisting of pairs $e_a(w)$, $e_b(w)$ for $0 \leq e_a(w) \leq 5$ and therefore $0 \leq e_b(w) \leq 20$, and the possible cyclically reduced words together with the representation and abstract structure of the corresponding quotient group.

We give an example of how we eleminate the words to obtain cyclically reduced ones for a given pair of numbers $e_a(w)$, $e_b(w)$.

Example 3.1. Let $e_a(w) = 4$ and $e_b(w) = 9$. Then the possible cyclically reduced words are

1) $ababab^3ab^4$ 6) $abab^2ab^4ab^2$
2) $ababab^4ab^3$ 7) $abab^2ab^3ab^3$
3) $abab^3abab^4$ 8) $abab^3ab^2ab^3$
4) $abab^2ab^2ab^4$ 9) $abab^3ab^3ab^2$
5) $abab^4ab^2ab^2$ 10) $ab^2ab^2ab^2ab^3$

We do not take the words such as $abab^3ab^4ab$, ab^3ab^4abab, $ab^4abababb^3$, which are all equivalent to the first one when equaled to identity.

Let us consider the first relation

$$ababab^3ab^4 = 1$$

together with existing ones $a^2 = b^5 = 1$. Writing $a = bababb^3ab^4$, one can obtain

$$1 = a^2 = bababb^3ab^4bababb^3ab^4 = bab^4ab^3$$

after cancelling out a^2's and b^5's. Multiplying both sides by b_4, we get

$$b^4 = ab^4ab^3$$

and after cancelling out b^3's, we have

$$b = ab^4a$$

which is equivalent to

$$aba = b^4$$

and finally we find $abab = (ab)^2 = 1$. Therefore the quotient group of $H(\frac{1+\sqrt{5}}{2})$ obtained by adding the first relation has a presentation

$$< a, b : \ a^2 = b^5 = (ab)^2 = 1 >$$

which is $T(2, 5, 2)$ isomorphic to the dihedral group D_5 of order 10.

For the remaining 9 words, similar operations give the same quotient group.

k	l	Possible cyclically reduced words	Representation of quotient group	Abstract structure of quotient group
0	1	b	$< a, b : a^2 = b^5 = b = 1 >$	C_2
0	2	b^2	$< a, b : a^2 = b^5 = b^2 = 1 >$	C_2
0	3	b^3	$< a, b : a^2 = b^5 = b^3 = 1 >$	C_2
0	4	b^4	$< a, b : a^2 = b^5 = b^4 = 1 >$	C_2
1	0	a	$< a, b : a^2 = b^5 = a = 1 >$	C_5
1	1	ab	$< a, b : a^2 = b^5 = ab = 1 >$	C_1
1	2	ab^2	$< a, b : a^2 = b^5 = ab^2 = 1 >$	C_1
1	3	ab^3	$< a, b : a^2 = b^5 = ab^3 = 1 >$	C_1
1	4	ab^4	$< a, b : a^2 = b^5 = ab^4 = 1 >$	C_1
2	2	$abab$	$< a, b : a^2 = b^5 = abab = 1 >$	D_5
2	3	$abab^2$	$< a, b : a^2 = b^5 = abab^2 = 1 >$	D_5
2	4	ab^2ab^2	$< a, b : a^2 = b^5 = ab^2ab^2 = 1 >$	D_5
		$abab^3$	$< a, b : a^2 = b^5 = abab^3 = 1 >$	D_5

2	5	ab^2ab^3	$< a, b : a^2 = b^5 = ab^2ab^3 = 1 >$	C_{10}
		$abab^4$	$< a, b : a^2 = b^5 = ab^2ab^4 = 1 >$	C_{10}
2	6	ab^2ab^4	$< a, b : a^2 = b^5 = ab^2ab^4 = 1 >$	D_5
		ab^3ab^3	$< a, b : a^2 = b^5 = ab^3ab^3 = 1 >$	D_5
2	7	ab^3ab^4	$< a, b : a^2 = b^5 = ab^3ab^4 = 1 >$	D_5
2	8	ab^4ab^4	$< a, b : a^2 = b^5 = ab^4ab^4 = 1 >$	D_5
3	3	$ababab$	$< a, b : a^2 = b^5 = ababab = 1 >$	A_5
3	4	$abababb^2$	$< a, b : a^2 = b^5 = abababb^2 = 1 >$	C_1
3	5	$ababab^3$	$< a, b : a^2 = b^5 = ababab^3 = 1 >$	C_5
		$abab^2ab^2$	$< a, b : a^2 = b^5 = abab^2ab^2 = 1 >$	C_5
3	6	$ab^2ab^2ab^2$	$< a, b : a^2 = b^5 = ab^2ab^2ab^2 = 1 >$	C_1
		$abababb^4$	$< a, b : a^2 = b^5 = abababb^4 = 1 >$	C_1
		$abab^2ab^3$	$< a, b : a^2 = b^5 = abab^2ab^3 = 1 >$	C_1
		$abab^3ab^2$	$< a, b : a^2 = b^5 = abab^3ab^2 = 1 >$	C_1
3	7	$abab^3ab^3$	$< a, b : a^2 = b^5 = abab^3ab^3 = 1 >$	C_1
		$ab^2ab^2ab^3$	$< a, b : a^2 = b^5 = ab^2ab^2ab^3 = 1 >$	C_1
		$abab^2ab^4$	$< a, b : a^2 = b^5 = abab^2ab^4 = 1 >$	C_1
		$abab^4ab^2$	$< a, b : a^2 = b^5 = abab^4ab^2 = 1 >$	C_1
3	8	$ab^2ab^2ab^4$	$< a, b : a^2 = b^5 = ab^2ab^2ab^4 = 1 >$	C_1
		$abab^3ab^4$	$< a, b : a^2 = b^5 = abab^3ab^4 = 1 >$	C_1
		$abab^4ab^3$	$< a, b : a^2 = b^5 = abab^4ab^3 = 1 >$	C_1
		$ab^2ab^3ab^3$	$< a, b : a^2 = b^5 = ab^2ab^3ab^3 = 1 >$	C_1
3	9	$abab^4ab^4$	$< a, b : a^2 = b^5 = abab^4ab^4 = 1 >$	C_1
		$ab^3ab^3ab^3$	$< a, b : a^2 = b^5 = ab^3ab^3ab^3 = 1 >$	C_1
		$ab^2ab^3ab^4$	$< a, b : a^2 = b^5 = ab^2ab^3ab^4 = 1 >$	C_1
		$ab^2ab^4ab^3$	$< a, b : a^2 = b^5 = ab^2ab^4ab^3 = 1 >$	C_1
3	10	$ab^2ab^4ab^4$	$< a, b : a^2 = b^5 = ab^2ab^4ab^4 = 1 >$	C_5
		$ab^3ab^3ab^4$	$< a, b : a^2 = b^5 = ab^3ab^3ab^4 = 1 >$	C_5
3	11	$ab^3ab^4ab^4$	$< a, b : a^2 = b^5 = ab^3ab^4ab^4 = 1 >$	C_1
3	12	$ab^4ab^4ab^4$	$< a, b : a^2 = b^5 = ab^4ab^4ab^4 = 1 >$	A_5

4	4	$abababab$	$< a, b : a^2 = b^5 = abababab = 1 >$	A_5
4	5	$abababab^2$	$< a, b : a^2 = b^5 = abababab^2 = 1 >$	C_{10}
4	6	$abababab^3$	$< a, b : a^2 = b^5 = abababab^3 = 1 >$	C_2
		$ababab^2ab^2$	$< a, b : a^2 = b^5 = ababab^2ab^2 = 1 >$	C_2
		$abab^2abab^2$	$< a, b : a^2 = b^5 = abab^2abab^2 = 1 >$	C_2
4	7	$abababab^4$	$< a, b : a^2 = b^5 = abababab^4 = 1 >$	D_5
		$abab^2ab^2ab^2$	$< a, b : a^2 = b^5 = abab^2ab^2ab^2 = 1 >$	D_5
		$ababab^2ab^3$	$< a, b : a^2 = b^5 = ababab^2ab^3 = 1 >$	D_5
		$ababab^3ab^2$	$< a, b : a^2 = b^5 = ababab^3ab^2 = 1 >$	D_5
		$abab^2abab^3$	$< a, b : a^2 = b^5 = abab^2abab^3 = 1 >$	D_5
4	8	$ababab^2ab^4$	$< a, b : a^2 = b^5 = ababab^2ab^4 = 1 >$	D_5
		$ababab^4ab^2$	$< a, b : a^2 = b^5 = ababab^4ab^2 = 1 >$	D_5
		$abab^2abab^4$	$< a, b : a^2 = b^5 = abab^2abab^4 = 1 >$	D_5
		$ababab^3ab^3$	$< a, b : a^2 = b^5 = ababab^3ab^3 = 1 >$	D_5
		$abab^3abab^3$	$< a, b : a^2 = b^5 = abab^3abab^3 = 1 >$	D_5
		$abab^2ab^2ab^3$	$< a, b : a^2 = b^5 = abab^2ab^2ab^3 = 1 >$	D_5
		$abab^2ab^3ab^2$	$< a, b : a^2 = b^5 = abab^2ab^3ab^2 = 1 >$	D_5
		$abab^3ab^2ab^2$	$< a, b : a^2 = b^5 = abab^3ab^2ab^2 = 1 >$	D_5
		$ab^2ab^2ab^2ab^2$	$< a, b : a^2 = b^5 = ab^2ab^2ab^2ab^2 = 1 >$	D_5
4	9	$ababab^3ab^4$	$< a, b : a^2 = b^5 = ababab^3ab^4 = 1 >$	D_5
		$ababab^4ab^3$	$< a, b : a^2 = b^5 = ababab^4ab^3 = 1 >$	D_5
		$abab^3abab^4$	$< a, b : a^2 = b^5 = abab^3abab^4 = 1 >$	D_5
		$abab^2ab^2ab^4$	$< a, b : a^2 = b^5 = abab^2ab^2ab^4 = 1 >$	D_5
		$abab^2ab^4ab^2$	$< a, b : a^2 = b^5 = abab^2ab^4ab^2 = 1 >$	D_5
		$abab^4ab^2ab^2$	$< a, b : a^2 = b^5 = abab^4ab^2ab^2 = 1 >$	D_5
		$abab^2ab^3ab^3$	$< a, b : a^2 = b^5 = abab^2ab^3ab^3 = 1 >$	D_5
		$abab^3ab^2ab^3$	$< a, b : a^2 = b^5 = abab^3ab^2ab^3 = 1 >$	D_5
		$abab^3ab^3ab^2$	$< a, b : a^2 = b^5 = abab^3ab^3ab^2 = 1 >$	D_5
		$ab^2ab^2ab^2ab^3$	$< a, b : a^2 = b^5 = ab^2ab^2ab^2ab^3 = 1 >$	D_5
4	10	$ababab^4ab^4$	$< a, b : a^2 = b^5 = ababab^4ab^4 = 1 >$	C_{10}

		$abab^4abab^4$	$< a, b : a^2 = b^5 = abab^4abab^4 = 1 >$	C_{10}
		$ab^2ab^2ab^2ab^4$	$< a, b : a^2 = b^5 = ab^2ab^2ab^2ab^4 = 1 >$	C_{10}
		$abab^3ab^3ab^3$	$< a, b : a^2 = b^5 = abab^3ab^3ab^3 = 1 >$	C_{10}
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		$abab^4ab^2ab^4$	$< a, b : a^2 = b^5 = abab^4ab^2ab^4 = 1 >$	D_5
		$abab^4ab^4ab^2$	$< a, b : a^2 = b^5 = abab^4ab^4ab^2 = 1 >$	D_5
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		$ab^2ab^2ab^4ab^3$	$< a, b : a^2 = b^5 = ab^2ab^2ab^4ab^3 = 1 >$	D_5
		$ab^2ab^3ab^2ab^4$	$< a, b : a^2 = b^5 = ab^2ab^3ab^2ab^4 = 1 >$	D_5
		$ab^2ab^3ab^3ab^3$	$< a, b : a^2 = b^5 = ab^2ab^3ab^3ab^3 = 1 >$	D_5
4	12	$abab^3ab^4ab^4$	$< a, b : a^2 = b^5 = abab^3ab^4ab^4 = 1 >$	D_5
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		$ab^2ab^4ab^2ab^4$	$< a, b : a^2 = b^5 = ab^2ab^4ab^2ab^4 = 1 >$	D_5
		$ab^3ab^3ab^3ab^3$	$< a, b : a^2 = b^5 = ab^3ab^3ab^3ab^3 = 1 >$	D_5
		$ab^2ab^3ab^3ab^4$	$< a, b : a^2 = b^5 = ab^2ab^3ab^3ab^4 = 1 >$	D_5
		$ab^2ab^3ab^4ab^3$	$< a, b : a^2 = b^5 = ab^2ab^3ab^4ab^3 = 1 >$	D_5
		$ab^2ab^4ab^3ab^3$	$< a, b : a^2 = b^5 = ab^2ab^4ab^3ab^3 = 1 >$	D_5

4	13	$abab^4ab^4ab^4$	$< a,b : a^2 = b^5 = abab^4ab^4ab^4 = 1 >$	D_5
		$ab^3ab^3ab^3ab^4$	$< a,b : a^2 = b^5 = ab^3ab^3ab^3ab^4 = 1 >$	D_5
		$ab^2ab^3ab^4ab^4$	$< a,b : a^2 = b^5 = ab^2ab^3ab^4ab^4 = 1 >$	D_5
		$ab^2ab^4ab^3ab^2$	$< a,b : a^2 = b^5 = ab^2ab^4ab^3ab^2 = 1 >$	D_5
		$ab^2ab^4ab^4ab^3$	$< a,b : a^2 = b^5 = ab^2ab^4ab^4ab^3 = 1 >$	D_5
4	14	$ab^2ab^4ab^4ab^4$	$< a,b : a^2 = b^5 = ab^2ab^4ab^4ab^4 = 1 >$	C_2
		$ab^3ab^3ab^4ab^4$	$< a,b : a^2 = b^5 = ab^3ab^3ab^4ab^4 = 1 >$	C_2
		$ab^3ab^4ab^3ab^4$	$< a,b : a^2 = b^5 = ab^3ab^4ab^3ab^4 = 1 >$	C_2
4	15	$ab^3ab^4ab^4ab^4$	$< a,b : a^2 = b^5 = ab^3ab^4ab^4ab^4 = 1 >$	C_{10}
4	16	$ab^4ab^4ab^4ab^4$	$< a,b : a^2 = b^5 = ab^4ab^4ab^4ab^4 = 1 >$	A_5
5	5	$abababab$	$< a,b : a^2 = b^5 = abababab = 1 >$	A_5
5	6	$abababab^2$	$< a,b : a^2 = b^5 = abababab^2 = 1 >$	C_2
5	7	$abababab^3$	$< a,b : a^2 = b^5 = abababab^3 = 1 >$	C_1
		$abababab^2ab^2$	$< a,b : a^2 = b^5 = abababab^2ab^2 = 1 >$	C_1
		$ababab^2abab^2$	$< a,b : a^2 = b^5 = ababab^2abab^2 = 1 >$	C_1
5	8	$abababab^4$	$< a,b : a^2 = b^5 = abababab^4 = 1 >$	C_2
		$abababab^2ab^3$	$< a,b : a^2 = b^5 = abababab^2ab^3 = 1 >$	C_2
		$ababab^2abab^3$	$< a,b : a^2 = b^5 = ababab^2abab^3 = 1 >$	C_2
		$abababab^3ab^2$	$< a,b : a^2 = b^5 = abababab^3ab^2 = 1 >$	C_2
		$ababab^3abab^2$	$< a,b : a^2 = b^5 = ababab^3abab^2 = 1 >$	C_2
		$ababab^2ab^2ab^2$	$< a,b : a^2 = b^5 = ababab^2ab^2ab^2 = 1 >$	C_2
		$abab^2abab^2ab^2$	$< a,b : a^2 = b^5 = abab^2abab^2ab^2 = 1 >$	C_2
5	9	$abababab^2ab^4$	$< a,b : a^2 = b^5 = abababab^2ab^4 = 1 >$	C_1
		$abababab^4ab^2$	$< a,b : a^2 = b^5 = abababab^4ab^2 = 1 >$	C_1
		$ababab^4abab^2$	$< a,b : a^2 = b^5 = ababab^4abab^2 = 1 >$	C_1
		$ababab^2abab^4$	$< a,b : a^2 = b^5 = ababab^2abab^4 = 1 >$	C_1
		$abababab^3ab^3$	$< a,b : a^2 = b^5 = abababab^3ab^3 = 1 >$	C_1
		$ababab^3abab^3$	$< a,b : a^2 = b^5 = ababab^3abab^3 = 1 >$	C_1
		$ababab^2ab^2ab^3$	$< a,b : a^2 = b^5 = ababab^2ab^2ab^3 = 1 >$	C_1
		$ababab^2ab^3ab^2$	$< a,b : a^2 = b^5 = ababab^2ab^3ab^2 = 1 >$	C_1

		$ababab^3ab^2ab^2$	$< a, b : a^2 = b^5 = ababab^3ab^2ab^2 = 1 >$	C_1
		$abab^3abab^2ab^2$	$< a, b : a^2 = b^5 = abab^3abab^2ab^2 = 1 >$	C_1
		$abab^2abab^3ab^2$	$< a, b : a^2 = b^5 = abab^2abab^3ab^2 = 1 >$	C_1
		$abab^2abab^2ab^3$	$< a, b : a^2 = b^5 = abab^2abab^2ab^3 = 1 >$	C_1
		$abab^2ab^2ab^2ab^2$	$< a, b : a^2 = b^5 = abab^2ab^2ab^2ab^2 = 1 >$	C_1
5	10	$abababab^3ab^4$	$< a, b : a^2 = b^5 = abababab^3ab^4 = 1 >$	C_5
		$abababab^4ab^3$	$< a, b : a^2 = b^5 = abababab^4ab^3 = 1 >$	C_5
		$ababab^4abab^3$	$< a, b : a^2 = b^5 = ababab^4abab^3 = 1 >$	C_5
		$ababab^3abab^4$	$< a, b : a^2 = b^5 = ababab^3abab^4 = 1 >$	C_5
		$ababab^2ab^3ab^3$	$< a, b : a^2 = b^5 = ababab^2ab^3ab^3 = 1 >$	C_5
		$ababab^3ab^2ab^3$	$< a, b : a^2 = b^5 = ababab^3ab^2ab^3 = 1 >$	C_5
		$ababab^3ab^3ab^2$	$< a, b : a^2 = b^5 = ababab^3ab^3ab^2 = 1 >$	C_5
		$abab^2abab^3ab^3$	$< a, b : a^2 = b^5 = abab^2abab^3ab^3 = 1 >$	C_5
		$abab^3abab^2ab^3$	$< a, b : a^2 = b^5 = abab^3abab^2ab^3 = 1 >$	C_5
		$abab^3abab^3ab^2$	$< a, b : a^2 = b^5 = abab^3abab^3ab^2 = 1 >$	C_5
		$ababab^4ab^2ab^2$	$< a, b : a^2 = b^5 = ababab^4ab^2ab^2 = 1 >$	C_5
		$ababab^2ab^4ab^2$	$< a, b : a^2 = b^5 = ababab^2ab^4ab^2 = 1 >$	C_5
		$ababab^2ab^2ab^4$	$< a, b : a^2 = b^5 = ababab^2ab^2ab^4 = 1 >$	C_5
		$abab^4abab^2ab^2$	$< a, b : a^2 = b^5 = abab^4abab^2ab^2 = 1 >$	C_5
		$abab^2abab^4ab^2$	$< a, b : a^2 = b^5 = abab^2abab^4ab^2 = 1 >$	C_5
		$abab^2abab^2ab^4$	$< a, b : a^2 = b^5 = abab^2abab^2ab^4 = 1 >$	C_5
		$abab^2ab^2ab^2ab^3$	$< a, b : a^2 = b^5 = abab^2ab^2ab^2ab^3 = 1 >$	C_5
		$abab^2ab^2ab^3ab^2$	$< a, b : a^2 = b^5 = abab^2ab^2ab^3ab^2 = 1 >$	C_5
		$abab^2ab^3ab^2ab^2$	$< a, b : a^2 = b^5 = abab^2ab^3ab^2ab^2 = 1 >$	C_5
		$abab^3ab^2ab^2ab^2$	$< a, b : a^2 = b^5 = abab^3ab^2ab^2ab^2 = 1 >$	C_5
		$ab^2ab^2ab^2ab^2ab^2$	$< a, b : a^2 = b^5 = ab^2ab^2ab^2ab^2ab^2 = 1 >$	C_5
5	11	$abababab^4ab^4$	$< a, b : a^2 = b^5 = abababab^4ab^4 = 1 >$	C_2
		$ababab^4abab^4$	$< a, b : a^2 = b^5 = ababab^4abab^4 = 1 >$	C_2
		$ababab^2ab^3ab^4$	$< a, b : a^2 = b^5 = ababab^2ab^3ab^4 = 1 >$	C_2
		$ababab^2ab^4ab^3$	$< a, b : a^2 = b^5 = ababab^2ab^4ab^3 = 1 >$	C_2

		$ababab^3ab^2ab^4$	$< a, b : a^2 = b^5 = ababab^3ab^2ab^4 = 1 >$	C_2
		$abababab^3ab^4ab^2$	$< a, b : a^2 = b^5 = abababab^3ab^4ab^2 = 1 >$	C_2
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		$abababab^4ab^3ab^2$	$< a, b : a^2 = b^5 = abababab^4ab^3ab^2 = 1 >$	C_2
		$abab^2abab^3ab^4$	$< a, b : a^2 = b^5 = abab^2abab^3ab^4 = 1 >$	C_2
		$abab^2abab^4ab^3$	$< a, b : a^2 = b^5 = abab^2abab^4ab^3 = 1 >$	C_2
		$abab^2ab^3abab^4$	$< a, b : a^2 = b^5 = abab^2ab^3abab^4 = 1 >$	C_2
		$abab^2ab^4abab^3$	$< a, b : a^2 = b^5 = abab^2ab^4abab^3 = 1 >$	C_2
		$abab^3abab^4ab^2$	$< a, b : a^2 = b^5 = abab^3abab^4ab^2 = 1 >$	C_2
		$abab^3ab^2abab^4$	$< a, b : a^2 = b^5 = abab^3ab^2abab^4 = 1 >$	C_2
		$abababab^3ab^3ab^3$	$< a, b : a^2 = b^5 = abababab^3ab^3ab^3 = 1 >$	C_2
		$abab^3abab^3ab^3$	$< a, b : a^2 = b^5 = abab^3abab^3ab^3 = 1 >$	C_2
		$abab^2ab^2ab^3ab^3$	$< a, b : a^2 = b^5 = abab^2ab^2ab^3ab^3 = 1 >$	C_2
		$abab^2ab^3ab^2ab^3$	$< a, b : a^2 = b^5 = abab^2ab^3ab^2ab^3 = 1 >$	C_2
		$abab^2ab^3ab^3ab^2$	$< a, b : a^2 = b^5 = abab^2ab^3ab^3ab^2 = 1 >$	C_2
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		$ab^2ab^2ab^2ab^2ab^3$	$< a, b : a^2 = b^5 = ab^2ab^2ab^2ab^2ab^3 = 1 >$	C_2
5	12	$abababab^2ab^4ab^4$	$< a, b : a^2 = b^5 = abababab^2ab^4ab^4 = 1 >$	C_1
		$abababab^4ab^2ab^4$	$< a, b : a^2 = b^5 = abababab^4ab^2ab^4 = 1 >$	C_1
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		$abab^4abab^4ab^2$	$< a, b : a^2 = b^5 = abab^4abab^4ab^2 = 1 >$	C_1
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		$abab^2ab^2ab^4ab^3$	$< a, b : a^2 = b^5 = abab^2ab^2ab^4ab^3 = 1 >$	C_1
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		$abab^4ab^2ab^2ab^3$	$< a, b : a^2 = b^5 = abab^4ab^2ab^2ab^3 = 1 >$	C_1
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		$ababab^3ab^3ab^4$	$< a, b : a^2 = b^5 = ababab^3ab^3ab^4 = 1 >$	C_1
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		$abab^3ab^3ab^2ab^3$	$< a, b : a^2 = b^5 = abab^3ab^3ab^2ab^3 = 1 >$	C_1
		$abab^3ab^3ab^3ab^2$	$< a, b : a^2 = b^5 = abab^3ab^3ab^3ab^2 = 1 >$	C_1
		$ab^2ab^2ab^2ab^3ab^3$	$< a, b : a^2 = b^5 = ab^2ab^2ab^2ab^3ab^3 = 1 >$	C_1
		$ab^2ab^2ab^3ab^2ab^3$	$< a, b : a^2 = b^5 = ab^2ab^2ab^3ab^2ab^3 = 1 >$	C_1
		$ab^2ab^2ab^2ab^2ab^4$	$< a, b : a^2 = b^5 = ab^2ab^2ab^2ab^2ab^4 = 1 >$	C_1
5	13	$abab^2ab^2ab^4ab^4$	$< a, b : a^2 = b^5 = abab^2ab^2ab^4ab^4 = 1 >$	C_1
		$abab^2ab^4ab^2ab^4$	$< a, b : a^2 = b^5 = abab^2ab^4ab^2ab^4 = 1 >$	C_1
		$abab^2ab^4ab^4ab^2$	$< a, b : a^2 = b^5 = abab^2ab^4ab^4ab^2 = 1 >$	C_1
		$abab^4ab^4ab^2ab^2$	$< a, b : a^2 = b^5 = abab^4ab^4ab^2ab^2 = 1 >$	C_1
		$abab^4ab^2ab^4ab^2$	$< a, b : a^2 = b^5 = abab^4ab^2ab^4ab^2 = 1 >$	C_1
		$abab^4ab^2ab^2ab^4$	$< a, b : a^2 = b^5 = abab^4ab^2ab^2ab^4 = 1 >$	C_1

		$ababab^3ab^4ab^4$	$< a, b : a^2 = b^5 = ababab^3ab^4ab^4 = 1 >$	C_1
		$abababab^4ab^3ab^4$	$< a, b : a^2 = b^5 = abababab^4ab^3ab^4 = 1 >$	C_1
		$abababab^4ab^4ab^3$	$< a, b : a^2 = b^5 = abababab^4ab^4ab^3 = 1 >$	C_1
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		$abab^3ab^4abab^4$	$< a, b : a^2 = b^5 = abab^3ab^4abab^4 = 1 >$	C_1
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		$abab^3ab^3ab^4ab^2$	$< a, b : a^2 = b^5 = abab^3ab^3ab^4ab^2 = 1 >$	C_1
		$abab^3ab^2ab^3ab^4$	$< a, b : a^2 = b^5 = abab^3ab^2ab^3ab^4 = 1 >$	C_1
		$abab^3ab^2ab^4ab^3$	$< a, b : a^2 = b^5 = abab^3ab^2ab^4ab^3 = 1 >$	C_1
		$abab^3ab^4ab^3ab^2$	$< a, b : a^2 = b^5 = abab^3ab^4ab^3ab^2 = 1 >$	C_1
		$abab^3ab^4ab^2ab^3$	$< a, b : a^2 = b^5 = abab^3ab^4ab^2ab^3 = 1 >$	C_1
		$abab^2ab^3ab^3ab^4$	$< a, b : a^2 = b^5 = abab^2ab^3ab^3ab^4 = 1 >$	C_1
		$abab^2ab^3ab^4ab^3$	$< a, b : a^2 = b^5 = abab^2ab^3ab^4ab^3 = 1 >$	C_1
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		$ab^2ab^2ab^2ab^4ab^3$	$< a, b : a^2 = b^5 = ab^2ab^2ab^2ab^4ab^3 = 1 >$	C_1
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		$ab^2ab^2ab^3ab^2ab^4$	$< a, b : a^2 = b^5 = ab^2ab^2ab^3ab^2ab^4 = 1 >$	C_1
		$ab^2ab^2ab^4ab^2ab^3$	$< a, b : a^2 = b^5 = ab^2ab^2ab^4ab^2ab^3 = 1 >$	C_1
5	14	$abab^4ab^4ab^2ab^3$	$< a, b : a^2 = b^5 = abab^4ab^4ab^2ab^3 = 1 >$	C_2
		$abab^4ab^4ab^3ab^2$	$< a, b : a^2 = b^5 = abab^4ab^4ab^3ab^2 = 1 >$	C_2
		$abab^4ab^2ab^4ab^3$	$< a, b : a^2 = b^5 = abab^4ab^2ab^4ab^3 = 1 >$	C_2
		$abab^4ab^2ab^3ab^4$	$< a, b : a^2 = b^5 = abab^4ab^2ab^3ab^4 = 1 >$	C_2
		$abab^4ab^3ab^4ab^2$	$< a, b : a^2 = b^5 = abab^4ab^3ab^4ab^2 = 1 >$	C_2

		$abab^4ab^3ab^2ab^4$	$< a, b : a^2 = b^5 = abab^4ab^3ab^2ab^4 = 1 >$	C_2
		$abab^2ab^4ab^4ab^3$	$< a, b : a^2 = b^5 = abab^2ab^4ab^4ab^3 = 1 >$	C_2
		$abab^2ab^4ab^3ab^4$	$< a, b : a^2 = b^5 = abab^2ab^4ab^3ab^4 = 1 >$	C_2
		$abab^2ab^3ab^4ab^4$	$< a, b : a^2 = b^5 = abab^2ab^3ab^4ab^4 = 1 >$	C_2
		$abab^3ab^4ab^4ab^2$	$< a, b : a^2 = b^5 = abab^3ab^4ab^4ab^2 = 1 >$	C_2
		$abab^3ab^4ab^2ab^4$	$< a, b : a^2 = b^5 = abab^3ab^4ab^2ab^4 = 1 >$	C_2
		$abab^3ab^2ab^4ab^4$	$< a, b : a^2 = b^5 = abab^3ab^2ab^4ab^4 = 1 >$	C_2
		$ababab^4ab^4ab^4$	$< a, b : a^2 = b^5 = ababab^4ab^4ab^4 = 1 >$	C_2
		$abab^4abab^4ab^4$	$< a, b : a^2 = b^5 = abab^4abab^4ab^4 = 1 >$	C_2
		$abab^3ab^3ab^3ab^4$	$< a, b : a^2 = b^5 = abab^3ab^3ab^3ab^4 = 1 >$	C_2
		$abab^3ab^3ab^4ab^3$	$< a, b : a^2 = b^5 = abab^3ab^3ab^4ab^3 = 1 >$	C_2
		$abab^3ab^4ab^3ab^3$	$< a, b : a^2 = b^5 = abab^3ab^4ab^3ab^3 = 1 >$	C_2
		$abab^4ab^3ab^3ab^3$	$< a, b : a^2 = b^5 = abab^4ab^3ab^3ab^3 = 1 >$	C_2
		$ab^2ab^2ab^2ab^4ab^4$	$< a, b : a^2 = b^5 = ab^2ab^2ab^2ab^4ab^4 = 1 >$	C_2
		$ab^2ab^2ab^4ab^2ab^4$	$< a, b : a^2 = b^5 = ab^2ab^2ab^4ab^2ab^4 = 1 >$	C_2
		$ab^2ab^2ab^3ab^3ab^4$	$< a, b : a^2 = b^5 = ab^2ab^2ab^3ab^3ab^4 = 1 >$	C_2
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		$ab^2ab^2ab^4ab^3ab^3$	$< a, b : a^2 = b^5 = ab^2ab^2ab^4ab^3ab^3 = 1 >$	C_2
		$ab^2ab^3ab^3ab^2ab^4$	$< a, b : a^2 = b^5 = ab^2ab^3ab^3ab^2ab^4 = 1 >$	C_2
		$ab^2ab^3ab^2ab^3ab^4$	$< a, b : a^2 = b^5 = ab^2ab^3ab^2ab^3ab^4 = 1 >$	C_2
		$ab^2ab^3ab^2ab^4ab^3$	$< a, b : a^2 = b^5 = ab^2ab^3ab^2ab^4ab^3 = 1 >$	C_2
		$ab^2ab^3ab^3ab^3ab^3$	$< a, b : a^2 = b^5 = ab^2ab^3ab^3ab^3ab^3 = 1 >$	C_2
5	15	$abab^2ab^4ab^4ab^4$	$< a, b : a^2 = b^5 = abab^2ab^4ab^4ab^4 = 1 >$	C_5
		$abab^4ab^2ab^4ab^4$	$< a, b : a^2 = b^5 = abab^4ab^2ab^4ab^4 = 1 >$	C_5
		$abab^4ab^4ab^2ab^4$	$< a, b : a^2 = b^5 = abab^4ab^4ab^2ab^4 = 1 >$	C_5
		$abab^4ab^4ab^4ab^2$	$< a, b : a^2 = b^5 = abab^4ab^4ab^4ab^2 = 1 >$	C_5
		$abab^3ab^3ab^4ab^4$	$< a, b : a^2 = b^5 = abab^3ab^3ab^4ab^4 = 1 >$	C_5
		$abab^3ab^4ab^3ab^4$	$< a, b : a^2 = b^5 = abab^3ab^4ab^3ab^4 = 1 >$	C_5
		$abab^3ab^4ab^4ab^3$	$< a, b : a^2 = b^5 = abab^3ab^4ab^4ab^3 = 1 >$	C_5
		$abab^4ab^4ab^3ab^3$	$< a, b : a^2 = b^5 = abab^4ab^4ab^3ab^3 = 1 >$	C_5

		$abab^4ab^3ab^4ab^3$	$< a, b : a^2 = b^5 = abab^4ab^3ab^4ab^3 = 1 >$	C_5
		$abab^4ab^3ab^3ab^4$	$< a, b : a^2 = b^5 = abab^4ab^3ab^3ab^4 = 1 >$	C_5
		$ab^2ab^2ab^3ab^4ab^4$	$< a, b : a^2 = b^5 = ab^2ab^2ab^3ab^4ab^4 = 1 >$	C_5
		$ab^2ab^2ab^4ab^3ab^4$	$< a, b : a^2 = b^5 = ab^2ab^2ab^4ab^3ab^4 = 1 >$	C_5
		$ab^2ab^2ab^4ab^4ab^3$	$< a, b : a^2 = b^5 = ab^2ab^2ab^4ab^4ab^3 = 1 >$	C_5
		$ab^2ab^3ab^2ab^4ab^4$	$< a, b : a^2 = b^5 = ab^2ab^3ab^2ab^4ab^4 = 1 >$	C_5
		$ab^2ab^4ab^2ab^3ab^4$	$< a, b : a^2 = b^5 = ab^2ab^4ab^2ab^3ab^4 = 1 >$	C_5
		$ab^2ab^4ab^3ab^2ab^4$	$< a, b : a^2 = b^5 = ab^2ab^4ab^3ab^2ab^4 = 1 >$	C_5
		$ab^2ab^4ab^3ab^3ab^3$	$< a, b : a^2 = b^5 = ab^2ab^4ab^3ab^3ab^3 = 1 >$	C_5
		$ab^2ab^3ab^4ab^3ab^3$	$< a, b : a^2 = b^5 = ab^2ab^3ab^4ab^3ab^3 = 1 >$	C_5
		$ab^2ab^3ab^3ab^4ab^3$	$< a, b : a^2 = b^5 = ab^2ab^3ab^3ab^4ab^3 = 1 >$	C_5
		$ab^2ab^3ab^3ab^3ab^4$	$< a, b : a^2 = b^5 = ab^2ab^3ab^3ab^3ab^4 = 1 >$	C_5
		$ab^3ab^3ab^3ab^3ab^3$	$< a, b : a^2 = b^5 = ab^3ab^3ab^3ab^3ab^3 = 1 >$	C_5
5	16	$abab^3ab^4ab^4ab^4$	$< a, b : a^2 = b^5 = abab^3ab^4ab^4ab^4 = 1 >$	C_1
		$abab^4ab^3ab^4ab^4$	$< a, b : a^2 = b^5 = abab^4ab^3ab^4ab^4 = 1 >$	C_1
		$abab^4ab^4ab^3ab^4$	$< a, b : a^2 = b^5 = abab^4ab^4ab^3ab^4 = 1 >$	C_1
		$abab^4ab^4ab^4ab^3$	$< a, b : a^2 = b^5 = abab^4ab^4ab^4ab^3 = 1 >$	C_1
		$ab^2ab^2ab^4ab^4ab^4$	$< a, b : a^2 = b^5 = ab^2ab^2ab^4ab^4ab^4 = 1 >$	C_1
		$ab^2ab^4ab^2ab^4ab^4$	$< a, b : a^2 = b^5 = ab^2ab^4ab^2ab^4ab^4 = 1 >$	C_1
		$ab^2ab^4ab^4ab^3ab^3$	$< a, b : a^2 = b^5 = ab^2ab^4ab^4ab^3ab^3 = 1 >$	C_1
		$ab^2ab^4ab^3ab^3ab^4$	$< a, b : a^2 = b^5 = ab^2ab^4ab^3ab^3ab^4 = 1 >$	C_1
		$ab^2ab^4ab^3ab^4ab^3$	$< a, b : a^2 = b^5 = ab^2ab^4ab^3ab^4ab^3 = 1 >$	C_1
		$ab^2ab^3ab^3ab^4ab^4$	$< a, b : a^2 = b^5 = ab^2ab^3ab^3ab^4ab^4 = 1 >$	C_1
		$ab^2ab^3ab^4ab^4ab^3$	$< a, b : a^2 = b^5 = ab^2ab^3ab^4ab^4ab^3 = 1 >$	C_1
		$ab^2ab^3ab^4ab^3ab^4$	$< a, b : a^2 = b^5 = ab^2ab^3ab^4ab^3ab^4 = 1 >$	C_1
		$ab^3ab^3ab^3ab^3ab^4$	$< a, b : a^2 = b^5 = ab^3ab^3ab^3ab^3ab^4 = 1 >$	C_1
5	17	$abab^4ab^4ab^4ab^4$	$< a, b : a^2 = b^5 = abab^4ab^4ab^4ab^4 = 1 >$	C_2
		$ab^3ab^3ab^3ab^4ab^4$	$< a, b : a^2 = b^5 = ab^3ab^3ab^3ab^4ab^4 = 1 >$	C_2
		$ab^3ab^3ab^4ab^3ab^4$	$< a, b : a^2 = b^5 = ab^3ab^3ab^4ab^3ab^4 = 1 >$	C_2
		$ab^2ab^3ab^4ab^4ab^4$	$< a, b : a^2 = b^5 = ab^2ab^3ab^4ab^4ab^4 = 1 >$	C_2

		$ab^2ab^4ab^3ab^4ab^4$	$< a, b : a^2 = b^5 = ab^2ab^4ab^3ab^4ab^4 = 1 >$	C_2
		$ab^2ab^4ab^4ab^3ab^4$	$< a, b : a^2 = b^5 = ab^2ab^4ab^4ab^3ab^4 = 1 >$	C_2
		$ab^2ab^4ab^4ab^4ab^3$	$< a, b : a^2 = b^5 = ab^2ab^4ab^4ab^4ab^3 = 1 >$	C_2
5	18	$ab^2ab^4ab^4ab^4ab^4$	$< a, b : a^2 = b^5 = ab^2ab^4ab^4ab^4ab^4 = 1 >$	C_1
		$ab^3ab^3ab^4ab^4ab^4$	$< a, b : a^2 = b^5 = ab^3ab^3ab^4ab^4ab^4 = 1 >$	C_1
		$ab^3ab^4ab^3ab^4ab^4$	$< a, b : a^2 = b^5 = ab^3ab^4ab^3ab^4ab^4 = 1 >$	C_1
5	19	$ab^3ab^4ab^4ab^4ab^4$	$< a, b : a^2 = b^5 = ab^3ab^4ab^4ab^4ab^4 = 1 >$	C_2
5	20	$ab^4ab^4ab^4ab^4ab^4$	$< a, b : a^2 = b^5 = ab^4ab^4ab^4ab^4ab^4 = 1 >$	A_5

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