December 11, Wednesday, 14:00 - 15:30

Lecture 1: Structure theorems, conjectures and questions of Iwasawa modules, past and present.

Abstract: We first discuss about Conjectures/Problems (in cyclotomic Iwasawa theory) Iwasawa described in his one of the final manuscripts (No.62 in Japanese and [U3] in English in his second volume of his collected papers). Then we indicate which of his problems has generalizations in more general settings of adjoint Selmer groups via the theory of modular forms. In the general case, the Iwasawa algebra is replaced by a local ring $\mathbb{T}$ of the p-adic Hecke algebra (which is a universal deformation ring by a theorem of Taylor–Wiles). We encounter new interesting problems related to Iwasawa's question in the general case. We study cyclicity questions in this series of lectures on adjoint Selmer groups. We fix a prime $p \geq 5$ throughout this lecture.

Note: In this first lecture, I make it as elementary as possible; so, basic knowledge of cyclotomic fields is sufficient. Though not a requirement, some knowledge of modular forms would help.

December 12, Thursday, 10:30 - 12:00

Lecture 2: Control of adjoint Selmer groups.

Abstract: Start with an odd 2-dimensional mod p Galois representation $\overline{\rho}$ into $\mathrm{GL}_2(\mathbb{F})$ for a finite field $\mathbb{F}$, we want to relate precisely the adjoint Selmer group of the universal ordinary representation to that of a given deformation of $\overline{\rho}$. By this, cyclicity is reduced to showing the $\mathbb{F}$-dimension of the adjoint Selmer group is less than or equal to 1. We exploit Kummer theory to prove this inequality when $\overline{\rho}$ comes from an Artin representation in the third lecture. Hereafter, we write W for the Witt vector ring with coefficients in $\mathbb{F}$ (i.e., the unique unramified discrete valuation ring flat over $\mathbb{Z}_p$ with residue field $\mathbb{F}$).

Note: Some basic facts from Galois cohomology help understanding this second lecture. For example, Long cohomology exact sequence, Inflation–restriction sequence, Shapiro's lemma. Also basic knowledge from algebraic geometry is helpful. For example, knowledge of the module of continuous Kähler differentials $\Omega_{R/A}$ for a p-profinite ring R over A and its relation to continuous derivation of the ring R over A.

December 13, Friday, 10:30 - 12:00

Lecture 3: Adjoint Selmer groups are cyclic.

Abstract: Assume that the mod p representation $\overline{\rho}$ in the second lecture is reduction modulo p of an irreducible odd Artin representation. We give a proof of cyclicity of the adjoint Selmer group as announced in the first lecture and existence of the algebraic adjoint p-adic L-function L such that the adjoint Selmer group is isomorphic to $\mathbb{T}/(L)$.

Note: Basic knowledge of algebraic number theory helps in this last lecture, though not a requirement. For example, Dirichlet's unit theorem, Local class field theory, Kummer theory and its relation to Galois cohomology and local Tate duality. Also some knowledge of commutative ring theory is helpful. For example, the second fundamental exact sequence of $\Omega_{R/A}$ and local complete intersection rings.