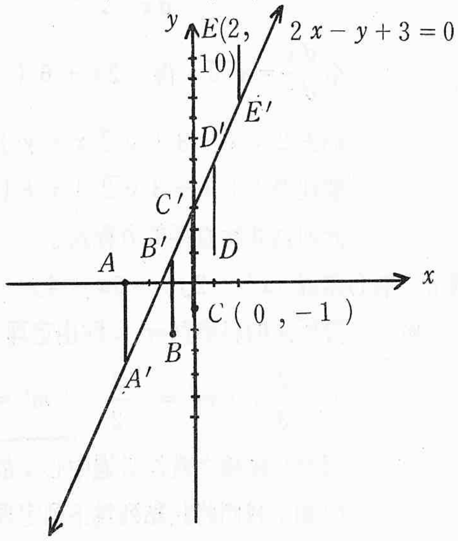
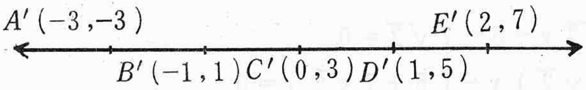
「應用數學」簡答

羅添壽　新化高中

1. A′(－3，－3)，B′(－1，1)，C′(0，3)，D′(1，5)，E′(2，7)
   ⟹ AA　＋BB′＋CC′＋DD′＋EE′＝17
   答：C



2. $A'(-3,-3)$　$E'(2,7)$

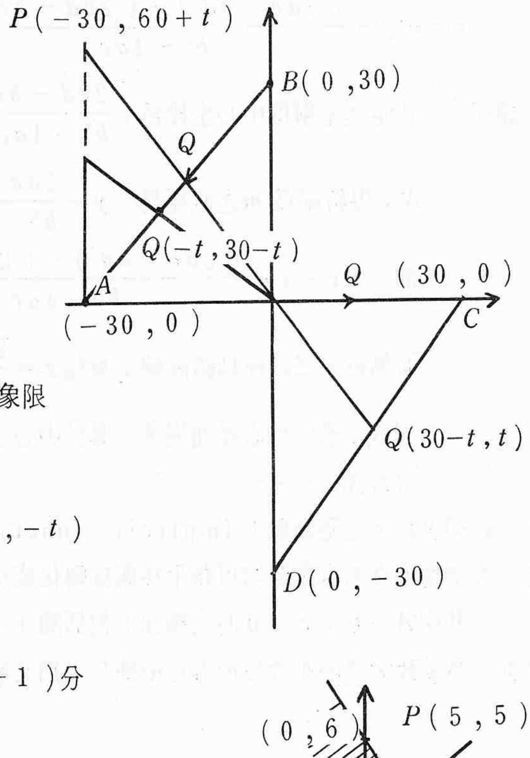


數線上之點　A′－B′－C′－D′－E′共五點，
∴　C′使至各點之距離和最小　∴　$x=0$，$y=3$

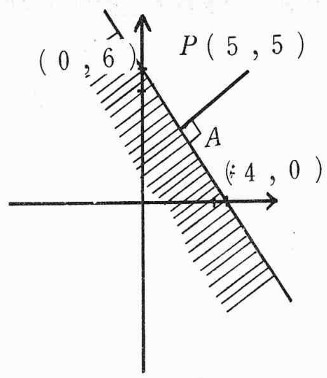
3. $f(x,y)=PA^2+PB^2+PC^2+PD^2+PE^2$
   $\Rightarrow x=-\dfrac{1}{5}$，$y=\dfrac{8}{5}$，$\therefore x-y=-\dfrac{9}{5}$

4. 解：令 t 分後 PQ 最短
   則 $P(-30,t)$，$Q(-t,30-t)$
   $\therefore\ PQ=\sqrt{(t-30)^2+(2t-30)^2}$
   當 $t=18$ 時，PQ 最短
   答：D

5. 解：① 設 t 分後第一次 P，Q，O 三點共線
   此時 Q 在第（Ⅱ）象限
   則 $P(-30,t)$　$Q(-t,30-t)$
   $\because\ m_{\overline{OP}}=m_{\overline{OQ}}\Rightarrow \dfrac{t}{-30}=\dfrac{30-t}{-t}$
   $\Rightarrow t=-15+15\sqrt{5}$

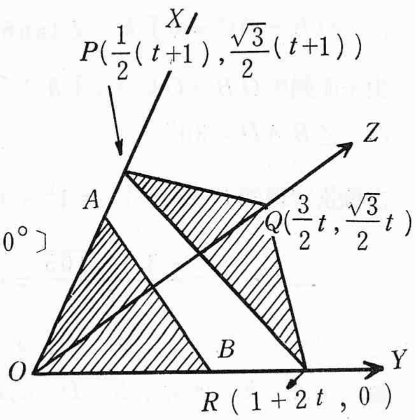


   ② 第二次 P，O，Q 三點共線此時 Q 在第Ⅳ象限
   當 Q 走至 C 所費時間為 $\left(30+\dfrac{60}{2}\right)$ 分
   $\therefore\ P(-30,60+t)$，$Q(30-t,-t)$
   $\because\ m_{\overline{OP}}=m_{\overline{OQ}}\Rightarrow \dfrac{60+t}{-30}=\dfrac{-t}{30-t}$
   $\Rightarrow t=\dfrac{-60+60\sqrt{3}}{2}=30(\sqrt{3}+1)$ 分
   答：CE



6. $\because\ f(x,y)=83-(x-5)^2-(y-5)^2$
   $=83-\left(\sqrt{(x-5)^2+(y-5)^2}\right)^2$ ……①
   欲使①式最大必 $\left(\sqrt{(x-5)^2+(y-5)^2}\right)^2$ 最小
   $\therefore\ PA^2=13$　$\therefore\ M=83-13=70$
   又由投影公式知 $A(2,3)$　即　$x_0=2$，$y_0=3$
   ∴　6. 答：B，D，E　7. 答：D

8. 設 t 秒後 P，Q，R 三點共線



今令 O 爲原點，則 $R(1+2t, 0)$

$\overrightarrow{OQ}=[\sqrt{3}\,t\cos 30^\circ, \sqrt{3}\,t\sin 30^\circ]$

$\therefore\ Q(\frac{3}{2}t, \frac{\sqrt{3}}{2}t)$

$\overrightarrow{OA}+\overrightarrow{AP}=[\cos 60^\circ, \sin 60^\circ]+[t\cos 60^\circ, t\sin 60^\circ]$

$\therefore\ P(\frac{1}{2}(t+1), \frac{\sqrt{3}}{2}(t+1))$

由 $m_{\overleftrightarrow{PR}}=m_{\overleftrightarrow{PQ}} \Rightarrow t=\frac{1+\sqrt{5}}{2}=2\cos 36^\circ$

答：C

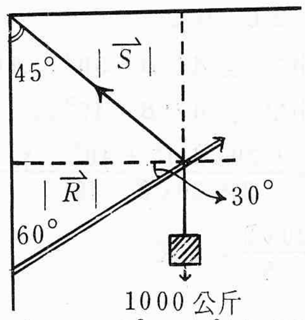
9. 解：

$$a\triangle PQR=\frac{1}{2}\begin{vmatrix}\frac{1}{2}(t+1) & \frac{\sqrt{3}}{2}(t+1) & 1\\ \frac{3}{2}t & \frac{\sqrt{3}}{2}t & 1\\ 1+2t & 0 & 1\end{vmatrix}=\frac{1}{2}\times 1\times 1\times\sin 60^\circ=a\triangle OAB$$

$\Rightarrow t^2-t-2=0 \Rightarrow t=2$

答：A，D

10. 由 $\begin{cases}|\overrightarrow{S}|\cos 45^\circ+|\overrightarrow{R}|\sin 30^\circ=1000\\ |\overrightarrow{S}|\sin 45^\circ=|\overrightarrow{R}|\cos 30^\circ\end{cases}$



$\Rightarrow |\overrightarrow{R}|=1000(\sqrt{3}-1)$ 公斤

$|\overrightarrow{S}|=\frac{\sqrt{3}}{\sqrt{2}}|\overrightarrow{R}|=500\sqrt{6}(\sqrt{3}-1)$ 公斤

答：A

11. $\overrightarrow{OA_3}=\overrightarrow{OA_1}=\overrightarrow{A_1A_2}+\overrightarrow{A_2A_3}$

$=[a, 0]+[ar\cos 120^\circ, ar\sin 120^\circ]+[ar^2\cos 240^\circ, ar^2\sin 240^\circ]$

$=[\frac{a(1-r)(2+r)}{2}, \frac{a(1-r)\sqrt{3}r}{2}]\quad\therefore\ p=2+r, q=\sqrt{3}r$

答：B，C

12. 考慮複數平面

$\overrightarrow{OA_n}=\overrightarrow{OA_1}+\overrightarrow{A_1A_2}+\overrightarrow{A_2A_3}+\cdots\cdots+\overrightarrow{A_{n-1}A_n}$

$=a+ar(\cos\frac{2}{3}\pi+i\sin\frac{2\pi}{3})+ar(\cos\frac{2}{3}\pi+i\sin\frac{2}{3}\pi)^2+\cdots\cdots$

$+ar^{n-1}(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3})^{n-1}$

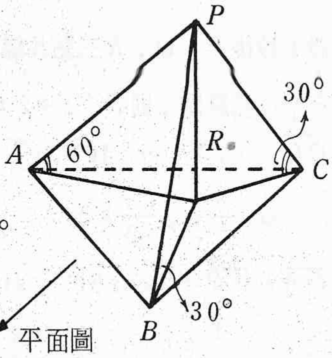
$\because\ |r(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3})|<1$

$\therefore\ \lim_{n\to\infty}\overrightarrow{OA_n}=\frac{a}{1-r(\cos\frac{2}{3}\pi+i\sin\frac{2}{3}\pi)}=\frac{a(2+r)}{2(1+r+r^2)}+\frac{\sqrt{3}ar}{2(1+r+r^2)}i$

答：B

13. $\tan 30^\circ=\frac{h}{OG}=\frac{h}{OB}=\frac{1}{\sqrt{3}}$

$\therefore\ OB = OC = \sqrt{3}\,h$ 又 $\tan 60° = \dfrac{h}{AO} \Rightarrow AO = \dfrac{h}{\sqrt{3}}$

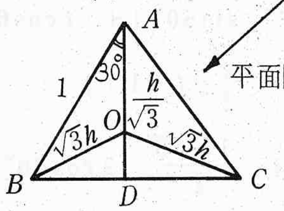


由平面圖中 $OB = OC = \sqrt{3}\,h$ ∴ O 落在 $\angle A$ 之平分線 $\overrightarrow{AD}$ 上

$\therefore\ \angle BAD = 30°$

由餘弦定理知 $(\sqrt{3}\,h)^2 = 1^2 + \left(\dfrac{h}{\sqrt{3}}\right)^2 - 2 \times 1 \times \dfrac{h}{\sqrt{3}} \cos 30°$

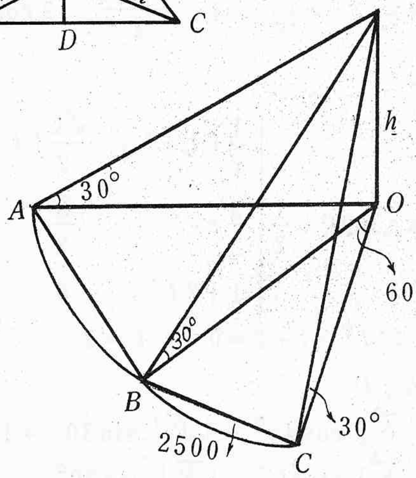
$$\Longrightarrow h = \frac{-3+\sqrt{105}}{16} \doteqdot 0.45$$



$\therefore\ \alpha = 0\ ,\ \beta = 4\ ,\ \gamma = 5$

答：①A，E　②A，B，D　③D

14 $\tan 30° = \dfrac{h}{BO} = \dfrac{h}{AO} = \dfrac{h}{CO} = \dfrac{1}{\sqrt{3}}$



$\therefore\ BO = AO = CO = \sqrt{3}\,h$

∴ 三點 A，B，C，D 以 O 為圓心共圓

又 $\angle BAC = 30° \Longrightarrow \angle BOC = 60°$

∴ $\triangle OBC$ 為正△

$\therefore\ \sqrt{3}\,h = 2500 \Longrightarrow h = 1445$

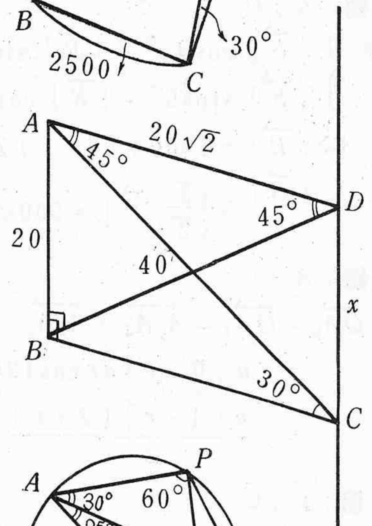
$\therefore\ p = 1\ ,\ q = 4\ ,\ r = 4$

答：①A　②C　③C

15. $\triangle ABC$ 中∵ $\angle ACB = 30°$ ∴ $AC = 40$

$\triangle ABD$ 中∵ $\angle ADB = 45°$ ∴ $AD = 20\sqrt{2}$

$$\cos 45° = \frac{(20\sqrt{2})^2 + 40^2 - x^2}{2 \times 20\sqrt{2} \times 40} \Longrightarrow x \doteqdot 20\sqrt{2}$$



$\therefore\ V = \dfrac{20\sqrt{2}}{5} = 4\sqrt{2}$

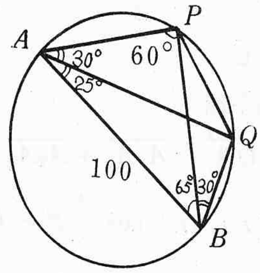
答：C

16. ∵ A，B，P，Q 四點共平面

且 $\angle PAQ = \angle PBQ = 30°$

∴ A，B，P，Q 四點共圓

$\therefore\ \triangle ABP$ 中 $\dfrac{100}{\sin 60°} = \dfrac{PB}{\sin 55°}$ …………①



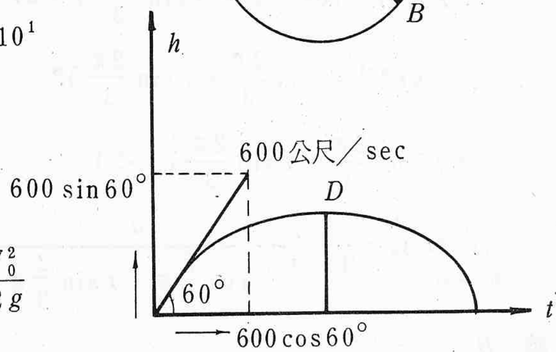
$\triangle PBQ$ 中 $\dfrac{PB}{\sin(180° - 55°)} = \dfrac{PQ}{\sin 30°}$ ………②

由①②得 $PQ = 57.7 \doteqdot 58 = (5.8) \times 10^1$

$\therefore\ a = 5\ ,\ b = 8\ ,\ m = 1$

答：①ACD　②BE　③AB

17. ① $h = V_0 t - \dfrac{1}{2} g t^2$



$$= -\frac{1}{2} g \left[t^2 - \frac{2V_0}{g} t + \frac{V_0^2}{g} \right] + \frac{V_0^2}{2g}$$

$$= -\frac{1}{2} g \left(t - \frac{V_0}{g} \right)^2 + \frac{V_0}{2g}$$

當 $t=\frac{V_0}{g}$ 時，最大高度為 $\frac{V_0^2}{2g}=\frac{(600\sin 60^\circ)^2}{2\times 9.8}=13776$ 公尺

② $2t=\frac{2V_0}{g}\doteqdot 106$

$\therefore$ 水平距離＝水平速度×時間＝31800 公尺

答：①A ②A

18. 解：如圖 $Q(\cos\theta,\sin\theta,0)$，$P(\cos 2\theta,\sin 2\theta,1)$

$\therefore\ PQ=\sqrt{(\cos 2\theta-\cos\theta)^2+(\sin 2\theta-\sin\theta)^2+1}$

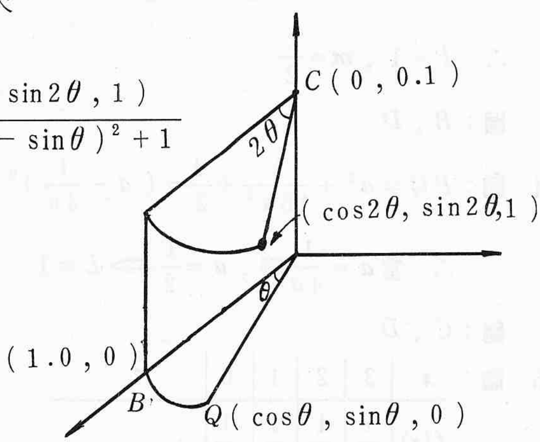
$=\sqrt{3-2\cos\theta}$

當 $\cos\theta=-1\Rightarrow Q(-1,0,0)$，

$P(1,0,1)$ 時，$M=\sqrt{5}$

當 $\cos\theta=1$ 即 $\theta=0\Rightarrow Q(1,0,0)$，

$P(1,0,1)$ 時，$m=1$

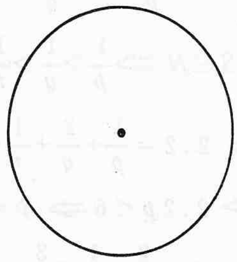


答：①BE ②$ABCD$

19. 解：$1^\circ=\frac{1}{360}$ 徑

$2\pi r\times\frac{1}{360}=2\times\frac{22}{7}\times 6370\times\frac{1}{360}=111$

$\therefore\ p=1,\ q=1,\ r=1$



答：①A ②A ③A

20. 解：$\because\ 25^\circ 18'-21^\circ 54'=3^\circ 24'\doteqdot 3.4^\circ$

$\because\ 111\times 3.4=377.4\ \therefore\ u=3,\ v=7,\ w=7$

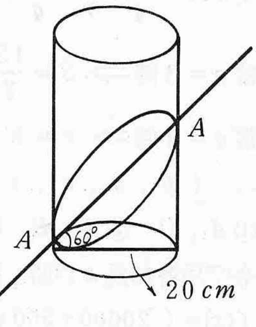
答：①A，C ②A，E ③A，E

21. 解：$\cos 60^\circ=\frac{20}{AA'}\Rightarrow AA'=40=2a\Rightarrow a=20$

又短軸長 $2b$ 即圓柱之底直徑

$\therefore\ 2b=20\Rightarrow b=10$

$\because\ c=\sqrt{400-100}=10\sqrt{3}\ \therefore\ e=\frac{\sqrt{3}}{2}$



答：①C ②B

22. 兩平面之交角為其法向角之夾角

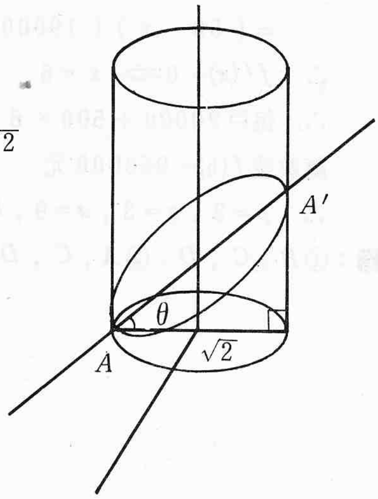
$\because\ \cos\theta=\frac{1}{1\times\sqrt{9+6+1}}=\frac{1}{4}$

$\therefore\ \cos\theta=\frac{2\sqrt{2}}{AA'}\Rightarrow AA'=2\sqrt{2}\times 4=2a\Rightarrow a=4\sqrt{2}$

又 $2b=2\sqrt{2}\Rightarrow b=\sqrt{2}$

$\therefore\ k=\pi ab=8\pi=25.1328=25$

$\therefore\ k=25=(2.5)\times 10^1\ \therefore\ a=2,\ b=5,\ m=1$



答：①B，D ②A，D ③A，C

23. $\because$ 光徑折射必過焦點 $(\frac{1}{4},0)$

$\therefore\ \overleftrightarrow{PF}:y=\frac{a}{a^2-\frac{1}{4}}(x-\frac{1}{4})$ 與 $y^2=x$ 聯立

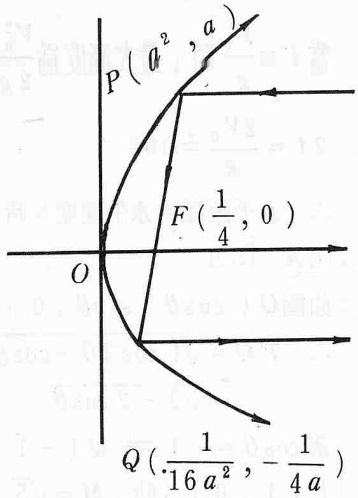
$\Longrightarrow 4ay^2-(4a^2-1)y-a=0$

$\Longrightarrow y=a$ 或 $y=-\dfrac{1}{4a}$

$\therefore\ PQ=\sqrt{(a^2+\dfrac{1}{16a^2}+\dfrac{1}{2})^2}=a^2+\dfrac{1}{16a^2}+\dfrac{1}{2}$

$\therefore\ l=1,\ m=\dfrac{1}{2}$

答：B，D



24. 解：$PQ=a^2+\dfrac{1}{16a^2}+\dfrac{1}{2}=(a-\dfrac{1}{4a})^2+1$

$\therefore$ 當 $a=\dfrac{1}{4a}$ 時，$a=\dfrac{1}{2}\Longrightarrow L=1$

答：C，D

25. 解：

x	3	2	1	0
$f(x)$	$\frac{1}{p}$	$\frac{1}{q}$	$\frac{1}{r}$	$\frac{1}{s}$

$\therefore 3\times\dfrac{1}{p}+2\times\dfrac{1}{q}+1\times\dfrac{1}{r}=2.2\ \because\ 1<p<q<r\in N$

且 $S\in N\Rightarrow\dfrac{1}{p}>\dfrac{1}{q}>\dfrac{1}{r}$

$\Rightarrow 2.2=\dfrac{3}{p}+\dfrac{2}{q}+\dfrac{1}{r}<\dfrac{3}{p}+\dfrac{3}{p}+\dfrac{3}{p}=\dfrac{6}{p}$

$\Rightarrow 2.2p<6\Rightarrow p=2$

又 $0.7=\dfrac{2}{q}+\dfrac{1}{r}<\dfrac{3}{q}\Rightarrow q=3$ 或 4

當 $q=3$ 時 $\Rightarrow S=\dfrac{15}{2}$ 不合

當 $q=4$ 時 $\Rightarrow r=5$，$S=20$

$\therefore\ (p,q,r,s)=(2,4,5,20)$

答：①A，D　②A，B　③C，E　④D，E

26. 解：令空戶增加爲 x 戶時，總收益最大，則

$f(x)=(20000+500x)(50-x)-1000(50-x)$

$=(50-x)(19000+500x)$

$\therefore\ f'(x)=0\Rightarrow x=6$

$\therefore$ 每戶 $20000+500\times6=23000$ 元時

總收益 $f(6)=968000$ 元

$\therefore\ p=2,\ q=3,\ a=9,\ b=6,\ c=8$

答：①B，C，D　②A，C，D