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PLANE SYMMETRIC VACUUM AND ZELDOVICH FLUID MODELS IN SELF CREATION THEORY

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Abstract. The problem of homogeneous plane symmetric perfect fluid distribution is considered in Barber's second self creation theory. Physically realistic vacuum and Zeldovich fluid models are obtained corresponding to two different cases. In the vacuum model, the Barber's scalar and the metric potentials behave alike and exhibit singularities at initial epoch and infinite future and past as well. The Zeldovich fluid model admits a Big-bang singularity at the initial epoch for both the modes of representation of scalar field when the coupling parametr $\lambda \in [-10^{-1}, 0)$.

1. Introduction. Two self creation cosmologies are proposed by Barber (1982) by modifying the Brans and Dicke (1961) theory and general relativity. These modified theories creat the universe out of self-contained gravitational and matter fields. Brans (1987) has pointed out that Barber's first theory is not only in disagreement with experiment but is actually inconsistent, in general. The second theory of Barber is a modification of general relativity to a variable G-theory and predicts local effects that are within the observational limits. In this theory the scalar field does not gravitate directly, but simply divides the matter tensor acting as a reciprocal gravitational constant. In the limit $\lambda \to 0$, this theory approaches standard relativity theory in every respect. Pimentel (1985) has solved the Friedmann-Barber field equations under the assumption of power law dependence of the

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scalar field on the scale factor, Soleng (1987 a,b) has generalised the work of Pimentel (1985) and obtained solutions for the vacuum dominated, radiation dominated and dustfilled Universes of the flat Friedmann-Robertson-Walker space-time. Reddy and Venkateswarlu (1989) have obtained Bianchi type VI_0 cosmological solutions both in vacuum and in presence of perfect fluid with pressure equal to energy density. Venkateswarlu and Reddy (1990) have also obtained spatially homogeneous and anisotropic Bianchi type-I cosmological models, when the source of gravitational field is a perfect fluid. Shanthi and Rao (1991) have also obtained spatially homogeneous and anisotropic Bianchi type II and III cosmological models in Barber's second self creation theory of gravitation both in vacuum and in presence of Zeldovich fluid. Here we propose to study the anisotropic homogeneous plane symmetric cosmological models in Barber's second self creation theory when the space time is described by a metric of the form

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2 dz^2 \tag{1.1}$$

Where A and B are functions of cosmic time only.

In section 2, we have derived the Barber-Einstein's field equations. Inorder to avoid the mathematical complexities due to the non-linear nature of the field equations we have considered two different cases in section 3 and found that the general fluid distribution degenerates vacuum model when A is constant and Zeldovich model when B constant.

In section 4, some physical properties of the models are discussed. The vacuum model exhibits singularities at initial epoch and infinite future and past as well. The Zeldovich fluid model admits a Big-bang singularity at the initial epoch when the coupling parameter $\lambda \in [-10^{-1}, 0)$. It is also seen that the model is expanding with increase of time but the rate of expansion becomes slow as time increases. The shape of the model changes uniformly both in x and y directions only. Also the Universe remains anisotropic throughout the evolution.

2. Barber-Einstein's Field Equations. The field eqns. of Barber's second self creation are

$$R_{ij} - (1/2)g_{ij}R = -8\pi\phi^{-1}T_{ij} \tag{2.1}$$

and

$$\Box\phi = (8/3)\pi\lambda T \tag{2.2}$$

Where λ is a coupling constant to be determined from experiment and ϕ is Barber's scalar. The measurements of the deflection of light restricts the value of coupling to $0 < |\lambda| < 10^{-1}$. In the limit $\lambda \to 0$, the theory approaches the standard general theory of relativity in every respect.

The energy-momentum tensor for perfect fluid is given by

$$T_{ij} = (p+\rho)v_i v_j - pg_{ij} \tag{2.3}$$

with

$$g^{ij}v_i v_j = 1 \tag{2.4}$$

where v^i is the four velocity vector of the fluid. p and ρ are isotropic pressure and energy-density respectively. With the help of eqns. (2.3) and (2.4), the field eqns. (2.1) and (2.2) for the metric (1.1) can be written in the following explicit form:

$$-8\pi\phi^{-1}p = A_{44}/A + A_4B_4/AB + B_{44}/B, \tag{2.5}$$

$$-8\pi\phi^{-1}p = 2A_{44}/A + A_4^2/A^2, \tag{2.6}$$

$$8\pi\phi^{-1}p = A_4^2/A^2 + 2A_4B_4/AB \tag{2.7}$$

and

$$(8/3)\pi\lambda(\rho - 3p) = \phi_{44} + \phi_4(2A_4/A + B_4/B). \tag{2.8}$$

Hereafterwards the subscript 4 after a field variable represents exact differentiation with respect to time.

3. Cosmological models. In this section, we intend to construct the cosmological models through the solution in Barber's second self creation cosmology. Inorder to overcome the underdeterminacy condition for determining five parameters from four eqns., we have considered here the following two cases with an addition condition.

Case-I: A=constant (Vacuum Model)

In this case the field eqns. (2.5) to (2.8) reduce to

$$-8\pi\phi^{-1}p = B_{44}/B, \tag{3.1}$$

$$-8\pi\phi^{-1}p = 0, \tag{3.2}$$

$$8\pi\phi^{-1}\rho = 0 \tag{3.3}$$

and

$$(8/3)\pi\lambda(\rho - 3p) = \phi_{44} + (B_4/B)\phi_4. \tag{3.4}$$

From eqns. (3.2) and (3.3), we obtain

$$p = \rho = 0, \text{ since } \phi^{-1} \neq 0 \tag{3.5}$$

Now eqns. (3.1) yields

$$B = \mathrm{d}_1 t + \mathrm{d}_2 \tag{3.6}$$

where d_1 and d_2 are arbitrary constants.

Using eqns. (3.5) and (3.6) in eqn. (3.4), we get

$$\phi = \mathrm{d}_0 \ln(\mathrm{d}_1 t + \mathrm{d}_2) + \mathrm{d}_3 \tag{3.7}$$

where d_0 and d_3 are integrating constants.

With suitable transformations the vacuum model in Barber's second self creation theory can be given in the form

$$\mathrm{d}s^2 = \mathrm{d}T^2 - (\mathrm{d}x^2 + \mathrm{d}y^2) - T^2\mathrm{d}z^2 \tag{3.8}$$

with the Barber's scalar

$$\phi = \mathrm{d}_0 \ln T + \mathrm{d}_4 \tag{3.9}$$

where d_4 is a constant. The model exhibits singularities at initial epoch and infinite future and past as well. It is obvious that both Barber's scalar and metric potential exhibit same singularities.

Case-II: B=constant (Zeldovich Fluid Model)

In this case the field eqns. (2.5)-(2.8) reduce to

$$-8\pi\phi^{-1}p = A_{44}/A, \tag{3.10}$$

$$-8\pi\phi^{-1}p = 2A_{44}/A + A_4^2/A^2, \tag{3.11}$$

$$8\pi\phi^{-1}\rho = A_4^2/A^2 \tag{3.12}$$

and

$$(8/3)\pi\lambda(\rho - 3p) = \phi_{44} + (A_4/A)\phi_4. \tag{3.13}$$

Using eqn. (3.10) in eqn. (3.11), we obtain

$$8\pi\phi^{-1}p = A_4^2/A^2 \tag{3.14}$$

Now eqn. (3.12) in eqn. (3.14) yield

$$p = \rho, \text{ since } \phi^{-1} \neq 0 \tag{3.15}$$

Eqn. (3.10) and eqn (3.11) yield a solution for A as

$$A^2 = k_1 t + k_2 \tag{3.16}$$

where k_1 and k_2 are integrating constants. Thus we find

$$8\pi\phi^{-1}p = 8\pi\phi^{-1}\rho = k_1^2/4(k_1 t + k_2)^2 \tag{3.17}$$

With the help of eqns. (3.16) and (3.17), eqn. (3.13) reduces to

$$(k_1 t + k_2)^2 \phi_{44} + k_1(k_1 t + k_2)\phi_4 + (\lambda/6)k_1^2 \phi = 0 \tag{3.18}$$

With a transformation $T \to t$, eqns. (3.17) and (3.18) reduce to

$$8\pi p = 8\pi \rho = \phi/(4T^2) \tag{3.19}$$

and

$$T^2 \phi_{TT} + T\phi_T + (\lambda/6)\phi = 0 \tag{3.20}$$

In view of this rescaled time we find the general solution of (3.20) in the form

$$\phi = \phi_1 + \phi_2 \tag{3.21}$$

where

$$\phi_1 = aT^{(\sqrt{-\lambda/6})} \tag{3.22}$$

and

$$\phi_2 = bT^{-(\sqrt{-\lambda/6})} \tag{3.23}$$

where a and b are arbitrary constants and for a physically realistic situation $\lambda \in [-10^{-1}, 0)$. Here we see that one of the modes of scalar field ϕ_1 is growing and the other one ϕ_2 is decaying. Now eqn. (3.17) gives the physical parameter pressure cum density as:

$$8\pi p = 8\pi \rho = (a/4)[T^{(\sqrt{(-\lambda/6)}-2)}] \tag{3.24}$$

or

$$8\pi p = 8\pi \rho = (b/4)[T^{-(\sqrt{(-\lambda/6)}+2)}] \tag{3.25}$$

In this case for both the modes the cosmological model for the Zeldovich Universe in Barber's second self creation theory can be given by

$$ds^2 = dT^2 - T(dx^2 + dy^2) - dz^2 \tag{3.26}$$

where the Barber's scalar is given by eqns. (3.22) and (3.23) and the corresponding parameters p and ρ are given by eqns. (3.24) and (3.25) respectively.

4. Discussion. It is interesting to note that in a plane symmetric space-time of the form (1.1), the general fluid distribution degenerates Vacuum model when A is constant and Zeldovich model when B is constant in Barber's second self creation theory. Here we conduct the physical study for the case with B=constant where matter field survives.

The scalar expansion θ calculated as

$$\theta = 1/T$$

from which it is evident that the Universe is expanding with increase of time but the rate of expansion becomes slow as time increases.

The shear scalar σ^2 for the model (3.26) is $\sigma^2 = 1/(6T^2)$. Since $\sigma^2 \to \infty$ as $T \to 0$ and $\sigma^2 \to 0$ as $T \to \infty$, the shape of the model changes uniformly in x and y-directions only and the rate of change of the shape of the Universe becomes slow with increase of time.

It is evident that $p(=\rho) \to 0$ as $T \to \infty$ and $p(=\rho) \to \infty$ as $T \to 0$, which indicates the presence of Big-bang singularity at initial epoch.

It has also been observed that $\lim_{T\to\infty} (\sigma/\theta) \approx 0.408$, which indicates that the Universe remains anisotropic throughout the evolution. By an indirect argument Collins et. al. (1980) have shown that the upper limit for σ/θ relating to isotropy of primordial black body radiation is 10^{-3}. However it is evident from the aforesaid analysis that the freedom is more in Barber's theory. For the second mode, the physical and kinematical parameters, and scalar field diverge at initial epoch and become zero as $T \to \infty$ whereas the scalar field behaves in opposite manner in the first mode of representation.

Also the rotation w and accleration $\dot{v}_i$ turn out to be zero. Thus the streamlines of the Zeldovich fluid are geodesics for both the modes of representation of scalar field. It is also interesting to note that when the Barber's coupling constant λ tends to zero, the corresponding Barber's scalar ϕ tends to constant. Subsequently the model (3.26) with eqns. (3.24) and (3.25) degenerates Zeldovich fluid model in Einstein theory. The Kretschmann curvature invariant is found to be,

$$L = 3/(16T^4)$$

which clearly indicates the presence of geometrical singularity at $T = 0$.

Both the energy conditions $\rho + 3p > 0$ and $\rho + p > 0$ are satisfied for $T > 0$. The model corresponding to non-vanished of both the metric potentials is under investigation and will be published later.

References

1. G. A. Barber, *General Relativity and Gravitation*, **14** (1982), 117.
2. C. Brans and R. H. Dicke, *Physics Review*, **124** (1961), 925.
3. C. Brans, *General Relativity and Gravitation*, **19** (1987), 949.
4. H. H. Soleng, *Astrophysics and Space Science*, **139** (1987a), 13.
5. H. H. Soleng, *Astrophysics and Space Science*, **102** (1987b), 67.
6. L. O. Pimentel, *Astrophysics and Space Science*, **116** (1985), 395.
7. D. R. K. Reddy and R. Venkateswarlu, *Astrophysics and Space Science*. **155** (1989), 135.
8. R. Venkateswarlu and D. R. K. Reddy, *Astrophysics and Space Science*, **168** (1990), 193.
9. K. Shanthi and V.U.M. Rao, *Astrophysics and Space Science*, **179** (1991), 147.

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