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ON AFFINE SKEW SYMMETRIC KILLING VECTOR FIELDS

BY

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Abstract. We study the conditions for a skew symmetric Killing vector field to be an affine vector field; we give explicit examples of manifolds realizing this situation.

1. Introduction. Let (M, g) be a Riemannian or pseudo-Riemannian manifold and let ∇ be the Levi-Civita connection on M. It is well known that Killing vector fields X (or infinitesimal isometries) play a distinguished role in differential geometry. They play also an important role when dealing with manifolds having indefinite metrics.

A vector field X which satisfies

$$\nabla X = X \wedge \mathcal{U} \Longleftrightarrow \mathcal{U}^\flat \otimes X - X^\flat \otimes \mathcal{U}, \tag{1}$$

with $\wedge$ the wedge product and $\mathcal{U}$ a torse forming vector field, is said to be a skew symmetric Killing vector field and $\mathcal{U}$ is called the generative of X [1] [2]. One finds that

$$dX^\flat = 2\mathcal{U}^\flat \wedge X^\flat \text{ and } \mathcal{L}_X X^\flat = 0,$$

where $\mathcal{L}$ is the Lie derivative, which shows that $X^\flat$ is an exterior recurrent form [3], having $2U^\flat$ as recurrence form, and also having the property to be a self-invariant form. Following a well-known property, X is an affine vector

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field if it satisfies

$$\mathcal{L}_X(\nabla X) = 0; \tag{2}$$

if M is compact and X is Killing, then following [16] the relation (2) is always satisfied.

Let then (M, g) be a paracompact or noncompact manifold. In the present paper we prove that the necessary and sufficient condition for a skew symmetric vector field X to be an affine vector field, is that the generative form ω of its associated torse forming vector field $\mathcal{U}$ be an exterior recurrent form having $\mathcal{U}^\flat$ as recurrence form. In this case, the existence of X is determined by an exterior differential system in involution, and M may be viewed as the local Riemannian product $M = M_X \times M^\perp$, such that

(1) M_X is a totally geodesic surface tangent to X and $\mathcal{U}$;

(2) $M_X^\perp$ is a 2-codimensional submanifold of umbilical (resp. geodesic) index 1.

Next, let σ be the volume element of M_X. If X defines an infinitesimal homothety of σ, then all vector fields of M_X are exterior concurrent, and by reference to [8], we conclude that M_X is a space form. In addition, it is shown in this case, that $U^\flat$ is a harmonic form. Finally, we give the following examples:

(1) X is carried by a $(2m+1)$-dimensional Kenmotsu manifold $M(\phi, \Omega, \eta, \xi, g)$ [17]. It is proved that in this case, in order for X to be affine, one necessarily must have that $\eta(X) =$ const.. Then, X, ϕX and ξ define a commutative triple and ϕX is also a skew symmetric Killing vector field. If $\mathbb{L} : \pi \to \pi \wedge \Omega$ means Weyl's (1.1) operator [4], then the following facts occur:

$$\mathcal{L}_X \Omega = 0, \qquad \mathcal{L}_{\phi X} \Omega = 0,$$

and

$$\mathcal{L}_X(\mathbb{L}X^\flat \wedge \Omega^q) = 0, \quad \mathcal{L}_{\phi X}(\mathbb{L}X^\flat \wedge \Omega^q) = 0.$$

Thus, X and ϕX define infinitesimal automorphisms of Ω and of all the $(2q+1)$-forms $X^\flat \wedge \Omega^q$.

(2) Let (M, g) be a space-time manifold having e_4 as time like vectorfield. If M carries two skew symmetric Killing vector fields X and Y, having e_4 as generative, then X and Y are affine Killing vector fields and M is a space form of curvature -1 [15].

2. Preliminaries. Let (M, g) be a Riemannian C^∞-manifold and let ∇ be the covariant differential operator defined by the metric tensor g. We assume that M is oriented and that ∇ is the Levi-Civita connection. Let ΓTM be the set of sections of the tangent bundle, and $\flat : TM \to T^*M$ the classical isomorphism (see also [5]) defined by g.

Next, following [5], we set

$$A^q(M, TM) = \mathrm{Hom}(\Lambda^q TM, TM),$$

and notice that the elements of $A^q(M, TM)$ are vector valued q-forms ($q <$ dim$M$). Denote by

$$d^\nabla : A^q(M, TM) \to A^{q+1}(M, TM)$$

the exterior covariant derivative operator with respect to ∇ (it should be noticed that in general $d^{\nabla^2} = d^\nabla \circ d^\nabla \neq 0$, unlike $d^2 = d \circ d = 0$). If $p \in M$ then the vector valued 1-form $dp \in A^1(M, TM)$ is the identity vector valued 1-form and is also called the soldering form of M [6]. Since ∇ is symmetric one has that $d^\nabla(dp) = 0$.

A vector field Z which satisfies

$$d^\nabla(\nabla Z) = \nabla^2 Z = \pi \wedge dp \in A^2(M, TM); \quad \pi \in \Lambda^1 M \tag{3}$$

is defined to be an exterior concurrent vector field [7] (see also [9]). The 1-form π is called the concurrence form and is defined by

$$\pi = \lambda Z^\flat, \quad \lambda \in \Lambda^0 M. \tag{4}$$

In this case, if $\mathcal{R}$ is the Ricci tensor of ∇, one has

$$\mathcal{R}(Z,Z) = -(n-1)\lambda g(Z,Z). \tag{5}$$

A vector field $\mathcal{U}$ whose covariant differential satisfies

$$\nabla \mathcal{U} = f\,dp + \omega \otimes \mathcal{U}; \quad f \in \Lambda^0 M, \tag{6}$$

is called a torse forming vector field [16] and the scalar f is the energy of $\mathcal{U}$[10]. We also remind Rosca's lemma [1]

$$d\mathcal{U}^\flat = \omega \wedge \mathcal{U}^\flat, \tag{7}$$

where ω is called the generative form of $\mathcal{U}$. A function $\mathbb{R}^n \to \mathbb{R}$ is isoparametric if $\|\nabla f\|^2$ and div ∇f are functions of f [11].

Let $\mathcal{O} = \{e_A | A = 1, \ldots n\}$ be a local field of orthonormal frames over M and let $\mathcal{O}^* = \text{covect}\{\omega^A\}$ be its associated coframe. Then the soldering form dp is expressed by

$$dp = \omega^A \otimes e_A; \quad A \in \{1, \ldots n\}; \tag{8}$$

and E. Cartans structure equations can be written in indexless manner as

$$\nabla e = \theta \otimes e, \tag{9}$$

$$d\omega = -\theta \wedge \omega, \tag{10}$$

$$d\theta = -\theta \wedge \theta + \Theta. \tag{11}$$

In the above equations θ (resp Θ) are the local connection forms in the tangent bundle TM (resp. the curvature 2-forms on M).

3. The general case. Let (M, g) be an n-dimensional Riemannian manifold with Levi-Civita connection ∇. Following [1], a skew symmetric Killing vector field X is defined by

$$\nabla X = X \wedge \mathcal{U} \Longleftrightarrow \mathcal{U}^\flat \otimes X - X^\flat \otimes \mathcal{U}, \tag{12}$$

($\wedge$: wedge product of vector fields) where $\mathcal{U}$, which is called the generating vector field of X, is a torse forming vector field [16], that is:

$$\nabla\mathcal{U} = f\,dp + \omega\otimes\mathcal{U}; \quad \omega\in\Lambda^1 M, \tag{13}$$

$f\in\Lambda^0 M$ is called the energy of $\mathcal{U}$. Recall also Rosca's lemma (7)

$$d\mathcal{U}^\flat = \omega\wedge\mathcal{U}^\flat$$

which shows that the dual form $\mathcal{U}^\flat$ of $\mathcal{U}$ is exterior recurrent [3] and has the generating form ω of $\mathcal{U}$ as recurrence form. Making use of the structure equations (10), one derives from (12) and (7) that

$$dX^\flat = 2\mathcal{U}^\flat\wedge X^\flat, \tag{14}$$

which means that $X^\flat$ is also an exterior recurrent form having $2\mathcal{U}^\flat$ as recurrence form. We notice that, in view of the fact that

$$X^\flat\wedge dX^\flat = 0$$

one may write locally $X^\flat = \tau_1 d\tau_2, \tau_1, \tau_2\in\Lambda^0 M$. Setting $s = g(X,\mathcal{U}), 2l_x = \|X\|^2$ ($2l_u = \|\mathcal{U}\|^2$energy of $\mathcal{U}$), one finds from (12) and (13) that

$$ds = s(\mathcal{U}^\flat + \omega) + (f - 2l_u)X^\flat, \tag{15}$$

$$dl_x = 2l_x\mathcal{U}^\flat - sX^\flat, \quad dl_U = (f\mathcal{U}^\flat + 2l_u\omega). \tag{16}$$

Then by (7), (15), and (16), one has

$$\mathcal{L}_X X^\flat = 0, \tag{17}$$

($\mathcal{L}$: Lie derivative) which sows that $X^\flat$ is a self invariant form. Now, following the well known definition, X is an affine vector field if it satisfies

$$\mathcal{L}_X\nabla X = 0, \tag{18}$$

and by (12), the above equation can be develloped as

$$\mathcal{L}_X\mathcal{U}^\flat\otimes X - \mathcal{L}_X\otimes\mathcal{U} - X^\flat\otimes\mathcal{L}_X\mathcal{U}. \tag{19}$$

By (12) and (13) one then sees that

$$\mathcal{L}_X\mathcal{U}^\flat = ds - s\omega + \omega(X)\mathcal{U}^\flat \tag{20}$$

and

$$\mathcal{L}_X\mathcal{U} = (f - 2l_u)X + (s + \omega(X))\mathcal{U}. \tag{21}$$

Since (19) holds for any X and $\mathcal{U}$, and taking account of (17), it follows that

$$f = 2l_u, \quad s + \omega(X) = 0. \tag{22}$$

Therefore, in order for (18) to hold, one must have that

$$ds - s\omega + \omega(X)\mathcal{U}^\flat = 0. \tag{23}$$

Further, by (22) and (23) one finally gets:

$$\frac{ds}{s} = \omega + \mathcal{U}^\flat \Rightarrow d\omega + d\mathcal{U}^\flat = 0, \tag{24}$$

i.e. $\mathcal{U}^\flat$ and $-\omega$ are homologous, and one may write

$$d\omega = \mathcal{U}^\flat \wedge \omega, \tag{25}$$

i.e. ω and $\mathcal{U}^\flat$ definde a reciprocal exterior recurrent pairing.

Denote by $D_X = \{X,\mathcal{U}\}$ the 2-distribution defined by X and $\mathcal{U}$, and by $\varphi = X^\flat \wedge \mathcal{U}^\flat$ its corresponding simple unit form. Since

$$i_X\varphi = 2l_x\mathcal{U}^\flat - sX^\flat, \quad i_\mathcal{U}\varphi = s\mathcal{U}^\flat - l_uX^\flat, \tag{26}$$

one can verify that for X', with $X' \in D_X$ one has $\nabla_X X' = 0$, which means that the distribution D_X is an autoparallel foliation and that the leaf M_X of D_X is a totally geodesic surface of M.

On the other hand, taking the exterior differential of φ, and using (13) and (14), one finds that

$$d\varphi = \omega \wedge \varphi, \tag{27}$$

i.e. φ is exterior concurrent and has ω as recurrence form. This agrees with the fact that the pairing $(X,\mathcal{U})$ defines a foliation, and consequently the complementary orthogonal distribution $D_X^\perp$ is also involutive. If we denote by $M_X^{\perp}$ the $(n-2)$-dimensional leaf of $D_X^{\perp}$, it is easily seen that $\mathcal{U}$ (resp. X) is a normal umbilical section (resp. a normal geodesic section) of $M^{\perp}$. Therefore M may be viewed as the local Riemannian product

$$M = M_X \times M_X^{\perp}$$

such that:

(i) M_X is a totally geodesic surface,

(ii) $M_X^{\perp}$ is a 2-codimensional submanifold of umbilical (resp. geodesic) index 1 (see [2]).

Let $\sum$ be the exterior differential system which determines X. Then by (13), (14), (16), (24), and (25), it follows that the characteristic numbers of $\sum$ are $r=6$, $s_0=3$, $s_1=3$. Since $r=s_0+s_1$ it follows that $\sum$ is in involution and by E. Cartan's test [12] we conclude that the existence of the affine skew symmetric Killing vector fild X is determined by 3 arbitrary functions of 1 argument.

We may formulate the:

Theorem 3.1. *The necessary and sufficient condition for a skew symmetric Killing vector field X on (M,g) to be an affine vector field, is that the generative form ω of its associated torse forming vector field $\mathcal{U}$ be an exterior recurrent form having $\mathcal{U}^\flat$ as recurrence form. The existence of such an X is determined by an exterior differential system in involution, and M may be viewed as the local Riemannian product $M = M_X \times M_X^{\perp}$, such that:*

(i) M_X is a totally geodesic surface tangent to X and $\mathcal{U}$;

(ii) $M_X^{\perp}$ is a 2-codimensional submanifold of umbilical (resp. geodesic) index 1.

4. A speical case. Since we found in section 3 that

$$i_X\varphi = 2l_x\mathcal{U}^\flat - sX^\flat, \quad s = g(X,\mathcal{U}), \tag{28}$$

one derives by (16), (22), and (24) that

$$\mathcal{L}_X\varphi = -s\varphi. \tag{29}$$

This means that X defines an infinitesimal conformal transformation of φ. Therefore, we will discuss in this section the case when X defines an infinitesimal homothety of φ, i.e. $s = g(X,\mathcal{U}) =$ const.. In this case, it follows from (16), (22), and (24) that

$$\mathcal{U}^\flat + \omega = 0 \Rightarrow d\mathcal{U}^\flat = 0, \quad f = \text{const.}. \tag{30}$$

Operating now on (12) and (13) by the operator d^∇, one derives

$$d^\nabla(\nabla X) = \nabla^2 X = f X^\flat \wedge dp \Rightarrow \mathcal{R}(X,Z) = -(n-1)fg(X,Z), \tag{31}$$

and

$$d^\nabla(\nabla \mathcal{U}) = \nabla^2 \mathcal{U} = f\mathcal{U}^\flat \wedge dp \Rightarrow \mathcal{R}(\mathcal{U},Z) = -(n-1)fg(\mathcal{U},Z), \tag{32}$$

where $\mathcal{R}$ denotes the Ricci tensor field of ∇ and $Z \in \Xi(M)$. Since the property of exterior concurrency is preserved by linearity and $f =$ const., it follows by (31) and (32) and reference to [9] that the surface M_X tangent to the distribution D_X is a space of curvature $-f$, i.e. up to a sign, the energy of $\mathcal{U}$.

Let

$$* : \Lambda^p T^* M \to \Lambda^{n-p} T^* M$$

be the star operator on M, and

$$\delta\pi = (-1)^{n(p+1)+1} * d * \pi$$

the codifferential of a q-form π. Taking the codifferential of $\mathcal{U}^\flat$, one obtains

$$*\mathcal{U}^\flat = \sum_{q=1}^{n} (-1)^{q-1} \mathcal{U}^q \omega^1 \wedge \cdots \wedge \hat{\omega}^q \wedge \cdots \wedge \omega^n, \tag{33}$$

and by reference to (13), where in stead of ω one has $\mathcal{U}^\flat$ (see (30)), one gets

$$\delta\mathcal{U}^\flat = - * d * \mathcal{U}^\flat = -(n-1)f. \tag{34}$$

But f being constant and $\mathcal{U}$ being closed, one may write

$$\Delta \mathcal{U}^\flat = d\delta \mathcal{U}^\flat = 0, \tag{35}$$

which proves the fact that $\mathcal{U}^\flat$ is a harmonic form.

Theorem 4.1. *Let X be a skew symmetric vector field on an n-dimensional Riemannian manifold M and let $\mathcal{U}$ be the associated torse forming vector field of X. Let also M_X be the surface tangent to X and $\mathcal{U}$, which foliates M. Then if X defines an infinitesimal homothety of the volume element of M_X, the energy of the torse forming vector field $\mathcal{U}$ is constant and:*

(i) *M_X is a space form of curvature $-f$,*

(ii) *$\mathcal{U}^\flat$ is a harmonic form.*

5. Examples.

5.1 On Kenmotsu manifolds. Let $M(\phi, \Omega, \eta, \xi, g)$ be a $(2m+1)$-dimensional Kenmotsu manifold. We recall that the quintuple $(\phi, \Omega, \eta, \xi, g)$ of structure tensors satisfy:

$$\begin{cases} \phi^2 & = -Id + \eta \otimes \xi, \ \phi\xi = 0, \ \eta(\xi) = 1, \\ (\phi Z)^\flat & = -\langle \phi dp, Z \rangle, \ g(\phi Z, Z') + g(Z, \phi Z') = 0, \\ (\nabla\phi)Z & = \eta(Z)\phi dp - (\phi Z)^\flat \otimes \xi \Longleftrightarrow (\nabla_{Z'}\phi)Z \\ & = -\eta(Z)\phi Z - g(Z, \phi Z')\xi, \\ \nabla\xi & = dp - \eta \otimes \xi; \end{cases} \tag{36}$$

and

$$\begin{cases} d\eta & = 0, \ d\Omega = 2\eta \wedge \Omega, \\ d^\nabla \phi dp & = 2\Omega \otimes \xi + \eta \wedge \phi dp. \end{cases} \tag{37}$$

(see also [8])

Remark. By the fourth equation of (36) and (37) one sees that ξ is a closed torse forming vector field.

Assume now that M carries a skew symmetric Killing affine vector field X having ξ as generative, that is

$$\nabla X = X \wedge \xi \Longleftrightarrow \nabla X = \eta \otimes X - X^\flat \otimes \xi, \ \mathcal{L}_X \nabla X = 0. \tag{38}$$

In view of (38), (14) now takes the form

$$dX^\flat = 2\eta \wedge X^\flat. \tag{39}$$

Then, making use of the structure equations (9) and (10), we derive by (38) that

$$dX^0 = 0; \quad \text{we have that} X^0 = \eta(X).) \tag{40}$$

Defining as usual (see also [13]), by

$$Z \to -i_Z\Omega,\ \Omega^\flat(Z) =^\flat Z,\ Z \in \Xi(M),$$

the symplectic isomorphism, one derives by (36)

$$d(\phi X)^\flat = 2\eta \wedge (\phi X)^\flat - 2X^0\Omega, \tag{41}$$

where

$$(\phi X)^\flat = -{}^\flat X. \tag{42}$$

Then taking the Lie derivative of Ω with respect to X one derives by (37) and (41) that

$$\mathcal{L}_X\Omega = 0. \tag{43}$$

Further, one also finds that

$${}^\flat(\phi X) = X^\flat - X^0\eta, \tag{44}$$

and making use of the structure equations (9) and (10), one also derives

$$\mathcal{L}_{\phi X}\Omega = 0. \tag{45}$$

Hence, (43) and (45) show that both X and ϕX define infinitesimal automorphisms of the structure 2-form Ω. Let now $\mathbb{L}$ be the Weyl operator of type (1.1) [4], acting on 1-forms, such that

$$\mathbb{L}\pi = \pi \wedge \Omega,\ \mathbb{L}^q\pi = \pi_q = \pi \wedge \Omega^q,\ \pi \in \Lambda^1 M. \tag{46}$$

Then, setting $(X^\flat)_q = X^\flat \wedge \Omega^q$, one derives on behalf of (39) and (40)

$$\mathcal{L}_X (X^\flat)_q = 0, \tag{47}$$

which says that X defines an infinitesimal automorphism of the $(2q+1)$-forms $X^\flat \wedge \Omega^q$. On the othere hand, with the help of (36) and (39), we obtain

$$\nabla \phi X = X^0 \phi dp + \phi X \wedge \xi. \tag{48}$$

Since the left inner product $\langle Z, \phi dp \rangle$ is equal to ϕZ, the expression for $\nabla \phi X$ reveals that ϕX is also a Killing vector field, having ξ as generative. Now, by (38), (48), and the fourth equation of (36), one finds:

$$[X, \phi X] = 0, \ [\xi, \phi X] = 0, \ [\xi, X] = 0. \tag{49}$$

This shows that $X, \phi X$, and ξ define a commutative triple (see also [14], [15]).

If we set as in Section 3, $2l_x = \|X\|^2$, then by (38) one gets

$$dl_x = 2l_x \eta - X^0 X^\flat, \tag{50}$$

and consequently

$$\nabla l_x = 2l_x \xi - X^0 X, \tag{51}$$

which gives

$$\|\nabla l_x\|^2 = 4l_x^2 - 2(X^0)^2 l_x. \tag{52}$$

Next, by the general formula $\mathrm{div} Z = \mathrm{tr} \nabla Z$ one finds that

$$\mathrm{div} \xi = 2m, \tag{53}$$

and

$$\mathrm{div}(\nabla l_x) = 2(m+1) l_x - 2(X^0)^2. \tag{54}$$

Then, since $X^0 = \text{const.}$, it follows from above, that $\|\nabla l_x\|^2$ and $\mathrm{div} l_x$ depend on l_x. Consequently, by reference to [11] (see also Section 2), one may say that the square of the length of a skew symmetric Killing vector field X on a Kenmotsu manifold $M(\phi, \Omega, \eta, \xi, g)$ is an isoparametric function. Summarizing, we have the following

Theorem 5.1. *Let $M(\phi, \Omega, \eta, \xi, g)$ be a $(2m+1)$-dimensioanl Kenmotsu manifolld. Then if X is a skew symmetric Killing vector field on M, having the Reeb vector field ξ as generative, one necessarily has that $\eta(X) = \text{const.}$, where $\eta = \xi^\flat$. In this case ϕX is also a Killing vector field and $X, \phi X$, and ξ define a commutative triple, i.e. $[X, \phi X] = 0$, $[\xi, \phi X] = 0$, $[\xi, X] = 0$. In addition, X and ϕX define infinitesimal automorphisms of the fundamental 2-form Ω and of all $(2q+1)$-forms $(X^\flat)_q = X^\flat \wedge \Omega$, i.e. $\mathcal{L}_X \Omega = 0$, $\mathcal{L}_{\phi X} \Omega = 0$, $\mathbb{L}(X^\flat)_q = 0$, $\mathcal{L}_{\phi X}(X^\flat)_q = 0$.*

5.2 On space-times. Let (M, g) be a general space-time and let X and Y be two skew symmetric Killing vector fields on M having the unit time like vector field e_4 on M as generative, then both X and Y are affine vector fields, and the existence of X and Y imply the following properties:

(i) M is a space form of cruvature -1,

(ii) the square of the length of X and Y are isoparametric functions,

(iii) the generalized Faraday form $\mathcal{F}$ on M is a relatively integral invariant of X and Y (see [15]).

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